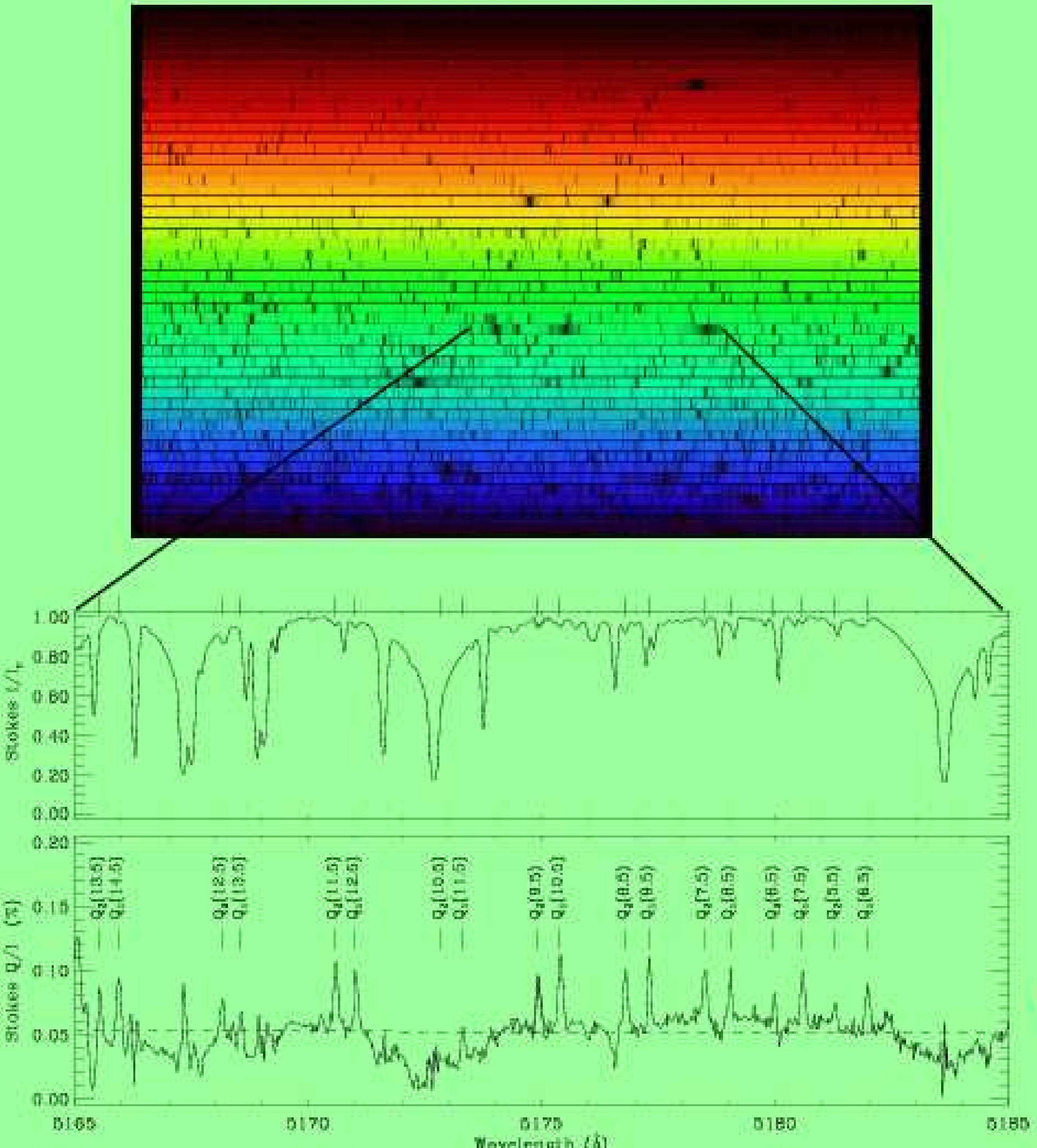
Effects of Magnetic Turbulence and Scattering on Line Polarization

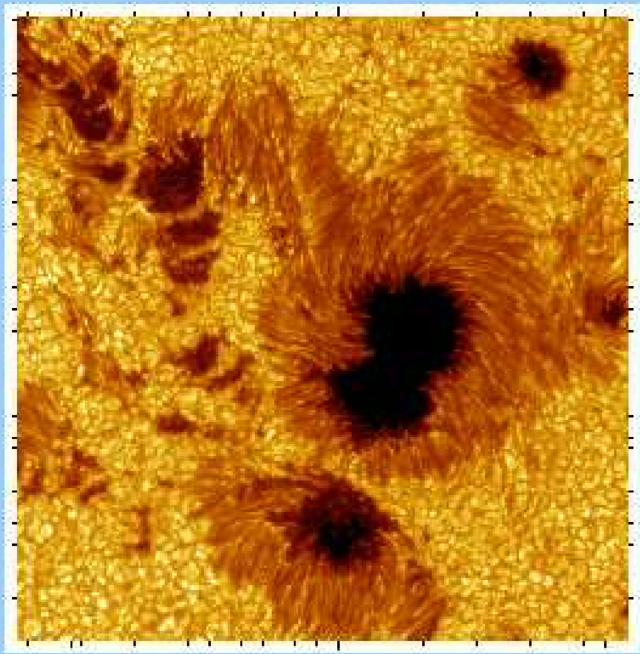
M. Sampoorna
CSIR Senior Research Fellow,
Joint Astronomy Program Student,
Indian Institute of Astrophysics, Bangalore 560 034, India

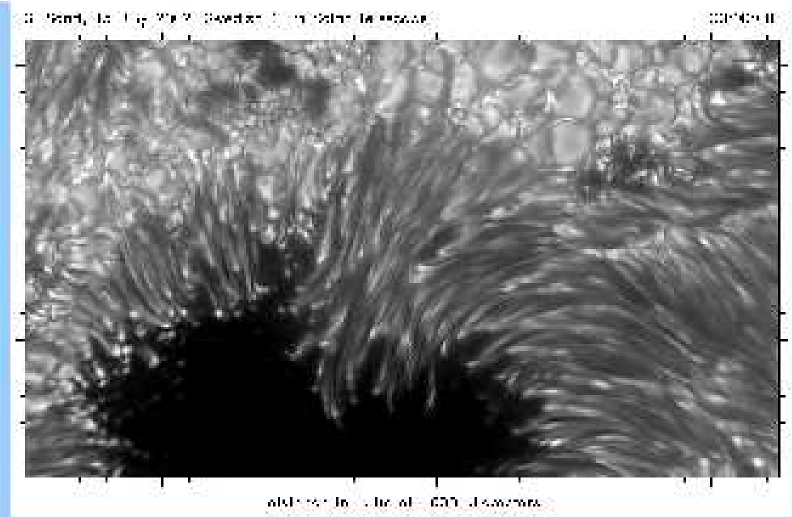


First solar spectrum

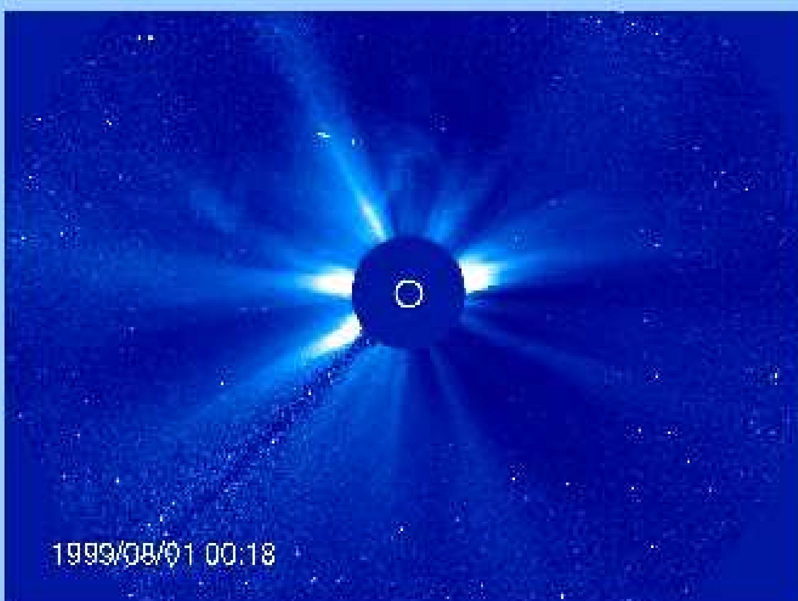
Second solar spectrum

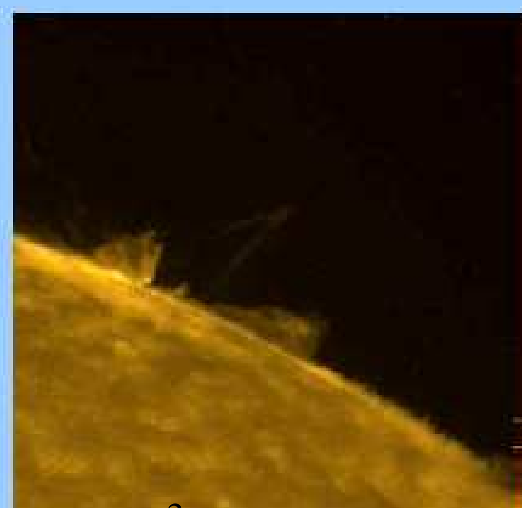
From Gandorfer's Atlas



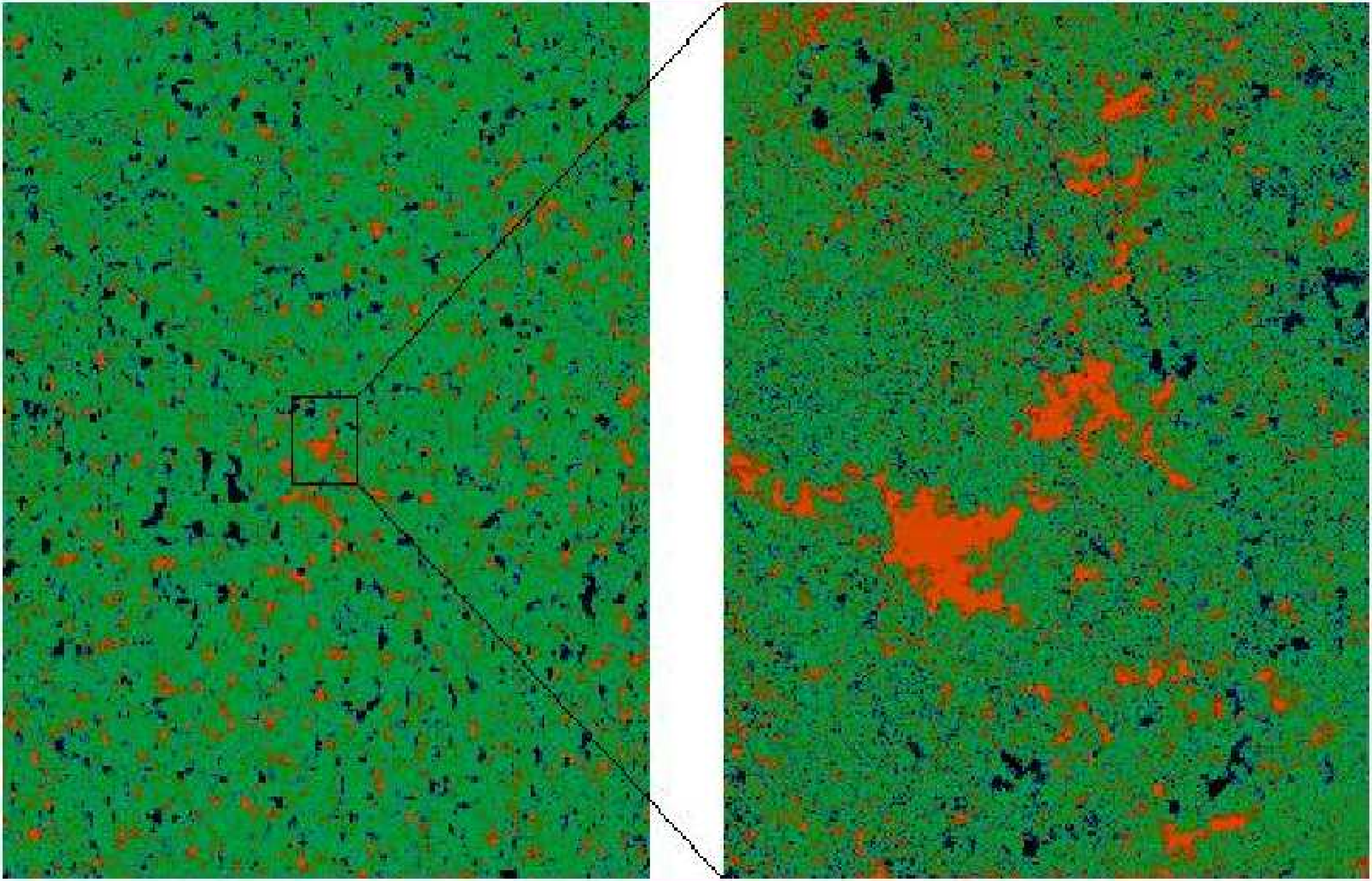


Gallery of magnetic structures



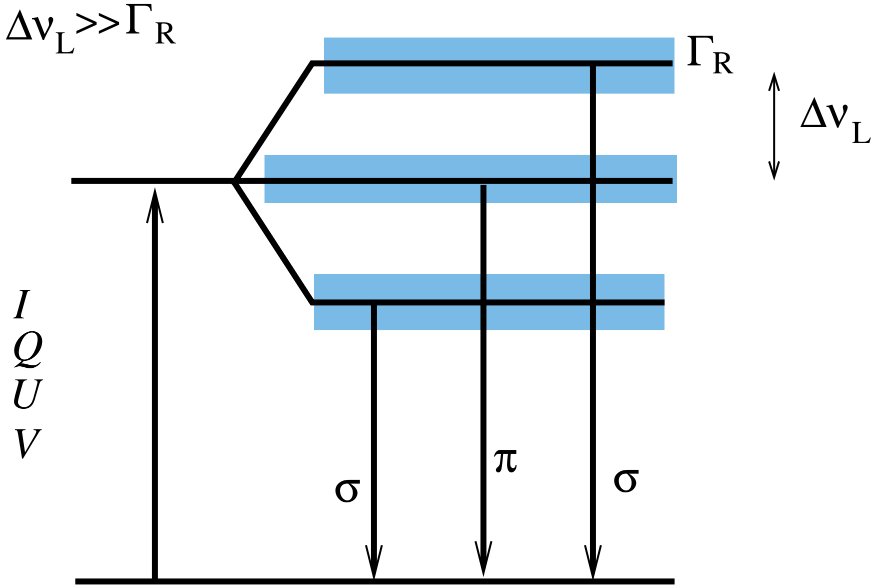


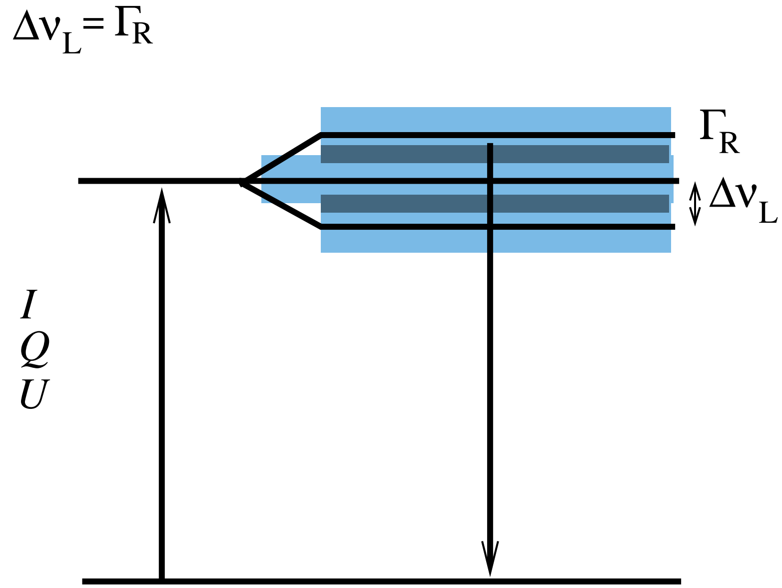
Fractal magnetic fields on the Sun



How to Study Solar magnetic fields?

- Two Physical Effects : Zeeman effect (pure absorption, LTE) and Hanle effect (scattering, NLTE)





Part I
Magnetic Turbulence

Introduction

- Observation and numerical simulation of magneto-convection show that the solar magnetic fields are turbulent in nature.
- Motivation : effect of a random magnetic field on spectral lines.
- Micro-turbulence : The **Mean Zeeman absorption matrix** is computed for various field vector distributions - correlation length $\ll$ photon mean free path
- Macro-turbulence : The **Mean Stokes intensity vector** - correlation length $\gg$ photon mean free path
- Meso-turbulence : **How to compute Mean Stokes parameters?** In this case two scales are comparable, $\Rightarrow$ More General theory is required.

Previous Work

- In Mihalas book (1978) one can find references to early work on the effects of velocity turbulence on unpolarized intensity.
- The problem of calculation of mean Zeeman absorption matrix in the micro-turbulent limit has been addressed fairly early by Dolginov & Pavlov (1972) and Domke & Pavlov (1979).
- For the general regime of meso-turbulence only few simple attempts are made (Landi Degl'Innocenti 1994). Therefore we take up this problem for a detailed study.

How to calculate the Mean Zeeman absorption matrix?

- Randomness of the magnetic field can be expressed in terms of a Probability Distribution Function (PDF) $P(\boldsymbol{B})d\boldsymbol{B}$

- Averaging process simply means performing the following integral :

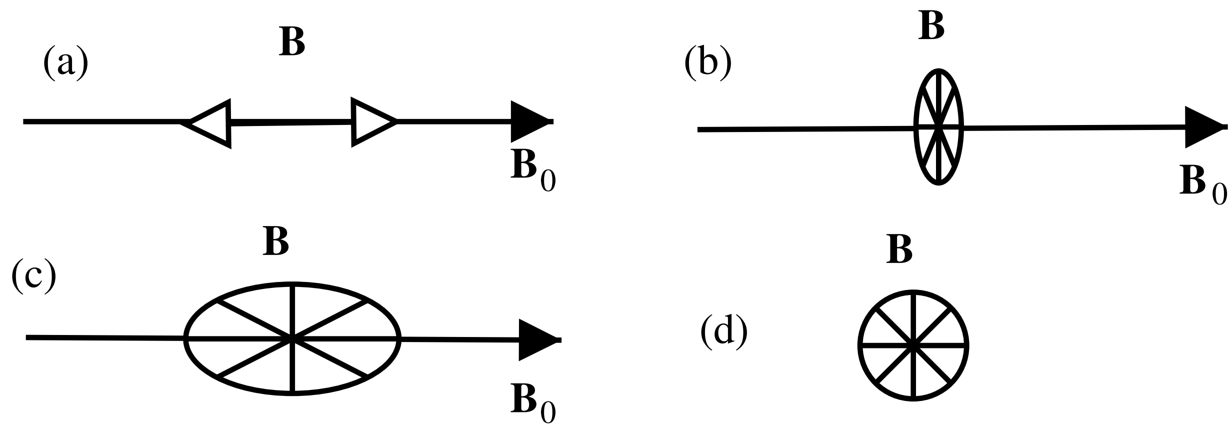
$$\langle \boldsymbol{\Phi} \rangle = \int \boldsymbol{\Phi}(\boldsymbol{B})\, P(\boldsymbol{B})\, d\boldsymbol{B}$$

  where $\boldsymbol{\Phi}$ is the Zeeman absorption matrix

- Performing the turbulent averaging is not a straight-forward process, since it is a triple integral which needs a careful numerical integration.

- The details of the analytic theory and the illustrations are presented in :- H. Frisch, M. Sampoorna, & K. N. Nagendra 2005, *Stochastic polarized line formation I. Zeeman propagation matrix in a random magnetic field*, A&A, 442, 11-28

Different Type of PDFs $P(\boldsymbol{B})d\boldsymbol{B}$



- (a) 1D turbulence : - (Ex : Flux tubes)

$$P_1(\boldsymbol{B})\,d\boldsymbol{B} = \frac{1}{(2\pi)^{1/2}\sigma}\exp\left[-\frac{(B-B_0)^2}{2\sigma^2}\right]\,dB,$$

where rms fluctuation $\sigma = [\langle (B-B_0)^2 \rangle]^{1/2}$.

- (b) 2D turbulence : - (Ex : Chromospheric Canopy)

$$P_2(\boldsymbol{B})\,d\boldsymbol{B} = \frac{1}{(2\pi)\,\sigma^2}\exp\left[-\frac{B_{\rm T}^2}{2\sigma^2}\right]\,B_{\rm T}\,dB_{\rm T}\,d\Psi, \quad 2\sigma^2 = \langle B_{\rm T}^2 \rangle,$$

where $B_{\rm T}$ is the random field perpendicular to $\boldsymbol{B}_0$, and Ψ is the field azimuth.

Different Type of PDFs Contd...

- 3D turbulence : - (Ex : Photospheric fields)

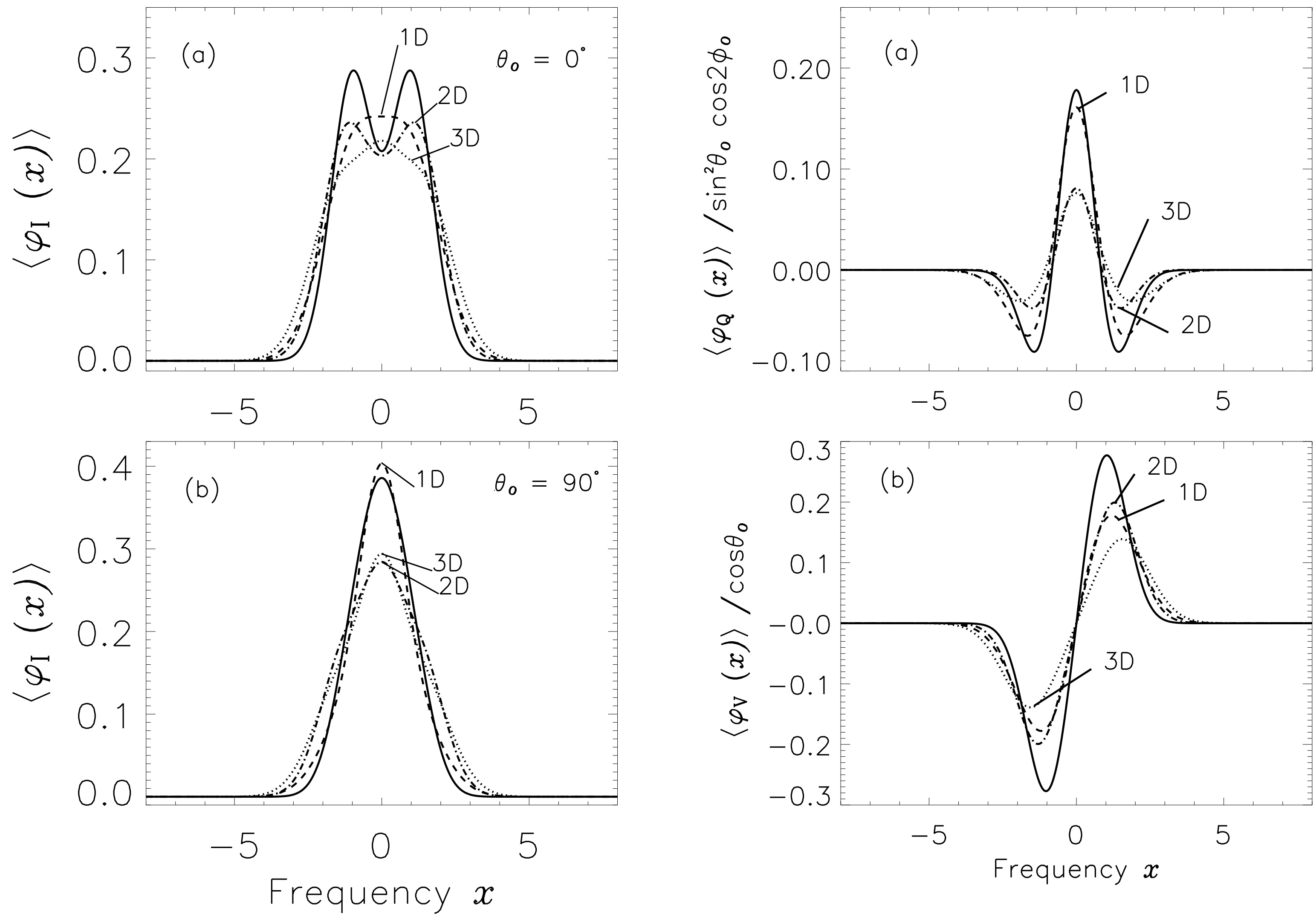
$$P_3(\boldsymbol{B})\, d\boldsymbol{B} = \frac{1}{(2\pi)^{3/2}\sigma^3} \exp\left[-\frac{(\boldsymbol{B}-\boldsymbol{B}_0)^2}{2\sigma^2}\right] B^2 \sin\Theta\, dB\, d\Theta\, d\Psi,$$

where $[\langle(\boldsymbol{B}-\boldsymbol{B}_0)^2\rangle]^{1/2} = \sqrt{3}\sigma$, Θ is field inclination.

- We have also recently considered PDFs of the form of Voigt, Stretched Exponentials, power law etc. (M. Sampoorna, K. N. Nagendra, H. Frisch, & J. O. Stenflo 2007, *Zeeman line formation in Solar magnetic fields : studies with empirical probability distribution functions*, A&A, Submitted)

- In the case of 3D turbulence $\langle\boldsymbol{\Phi}\rangle$ can be expressed in terms of generalized Voigt and Faraday-Voigt functions. They were first introduced by Dolginov & Pavlov (1972). We have studied properties of these functions and derived simple recurrence relations to compute them as well as their derivatives. (M. Sampoorna, K. N. Nagendra, & H. Frisch 2007, *Generalized Voigt functions and their derivatives*, JQSRT, 104, 71-85)

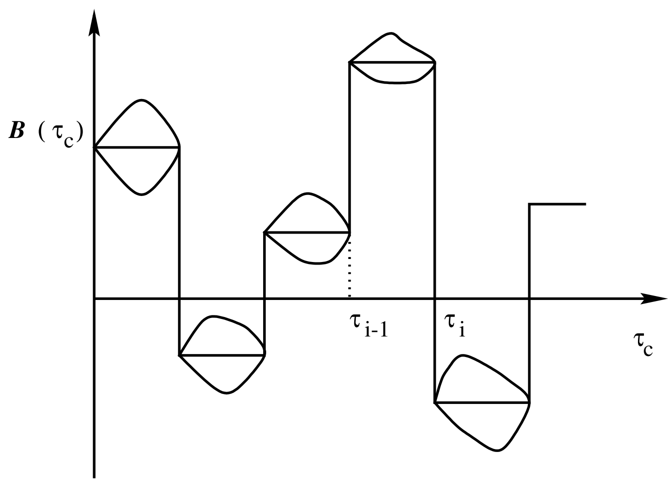
Polarized mean opacities for different PDFs



Freq. x: in Doppler width units $\Delta_D = \frac{\nu_0}{c}\sqrt{v_{th}^2 + v_{turb}^2}$; mean field $\boldsymbol{B}_0(|\boldsymbol{B}|_0, \theta_0, \phi_0)$; Zeeman shifts by the mean field and rms fluctuations $\Delta|\boldsymbol{B}|_0 = 1.$, $\Delta\sigma = 1./\sqrt{2}$; $\Delta = \frac{ge}{4\pi mc}\frac{1}{\Delta_D}$; left panels : $\langle\varphi_I\rangle$ for $\theta_0 = 0°$ and $\theta_0 = 90°$; right panels $\langle\varphi_Q\rangle$ and $\langle\varphi_V\rangle$

Model for a random field with 'finite' correlation length

- We represent fluctuating magnetic field by a Kubo-Anderson process (KAP).
- Properties of KAP : Markovian; stationary; piece-wise constant; characterized by a correlation length and a PDF.
- KAP describes the atmosphere as a number of "eddies".



- In each eddy the field is constant and its value drawn at random from a PDF. Further it "jumps" at the boundaries of the eddies.
- τ_i are jumping points distributed acc. to Poisson law with no. of jumps/unit optical depth = ν. "correlation length" or $\simeq$ size of the Eddy $= 1/\nu$.

$\Rightarrow$ limit $\nu \to 0$: macro-turbulence

$\Rightarrow$ limit $\nu \to \infty$: micro-turbulence

Stochastic Radiative Transfer Equation

- Transfer equation for the Stokes parameters $\boldsymbol{I}(I, Q, U, V)$

$$d\boldsymbol{I}/d\tau_c = (\mathbf{E} + \beta\mathbf{\Phi})[\boldsymbol{I} - \boldsymbol{S}]$$

$\mathbf{E}$: 4×4 unit matrix, $\mathbf{\Phi}$: Zeeman line absorption matrix, τ_c : continuum optical depth

- In Milne-Eddington (ME) model $\beta = \kappa_{\text{line}}/\kappa_{\text{cont}} =$ constant (namely opacities remaining constant). For ME model source vector $\boldsymbol{S} = (C_0 + C_1\tau_c)\boldsymbol{U}$, with $\boldsymbol{U} = (1, 0, 0, 0)^{\text{T}}$. Therefore possible to solve transfer equation analytically.

- Residual emergent Stokes parameters (at $\tau_c = 0$)

$$\boldsymbol{r}(0) = \frac{1}{C_1}[\boldsymbol{I}_c(0) - \boldsymbol{I}(0)]$$

where $\boldsymbol{I}_c(0)$: continuum intensity

Mean residual emergent Stokes parameters

- General expression for the mean value

$$\langle \boldsymbol{r}(0)\rangle_{\mathrm{KAP}} = (1+\nu)\mathbf{R}_{\mathrm{macro}}\left(\frac{\beta}{1+\nu}\boldsymbol{\Phi}\right)\left[\mathbf{E}+\nu\mathbf{R}_{\mathrm{macro}}\left(\frac{\beta}{1+\nu}\boldsymbol{\Phi}\right)\right]^{-1}\boldsymbol{U}$$

with

$$\mathbf{R}_{\mathrm{macro}}\left(\frac{\beta}{1+\nu}\boldsymbol{\Phi}\right) = \left\langle \frac{\beta}{1+\nu}\boldsymbol{\Phi}\left[\mathbf{E}+\frac{\beta}{1+\nu}\boldsymbol{\Phi}\right]^{-1}\right\rangle_{P(\boldsymbol{B})}$$

$1/\nu$: correlation length in optical unit of the continuum

- Micro and macro-turbulent limits :

$$\langle \boldsymbol{r}(0)\rangle_{\mathrm{micro}} = \beta\langle\boldsymbol{\Phi}\rangle[\mathbf{E}+\beta\langle\boldsymbol{\Phi}\rangle]^{-1}\boldsymbol{U}$$

$$\langle \boldsymbol{r}(0)\rangle_{\mathrm{macro}} = \langle\beta\boldsymbol{\Phi}[\mathbf{E}+\beta\boldsymbol{\Phi}]^{-1}\rangle\boldsymbol{U}$$

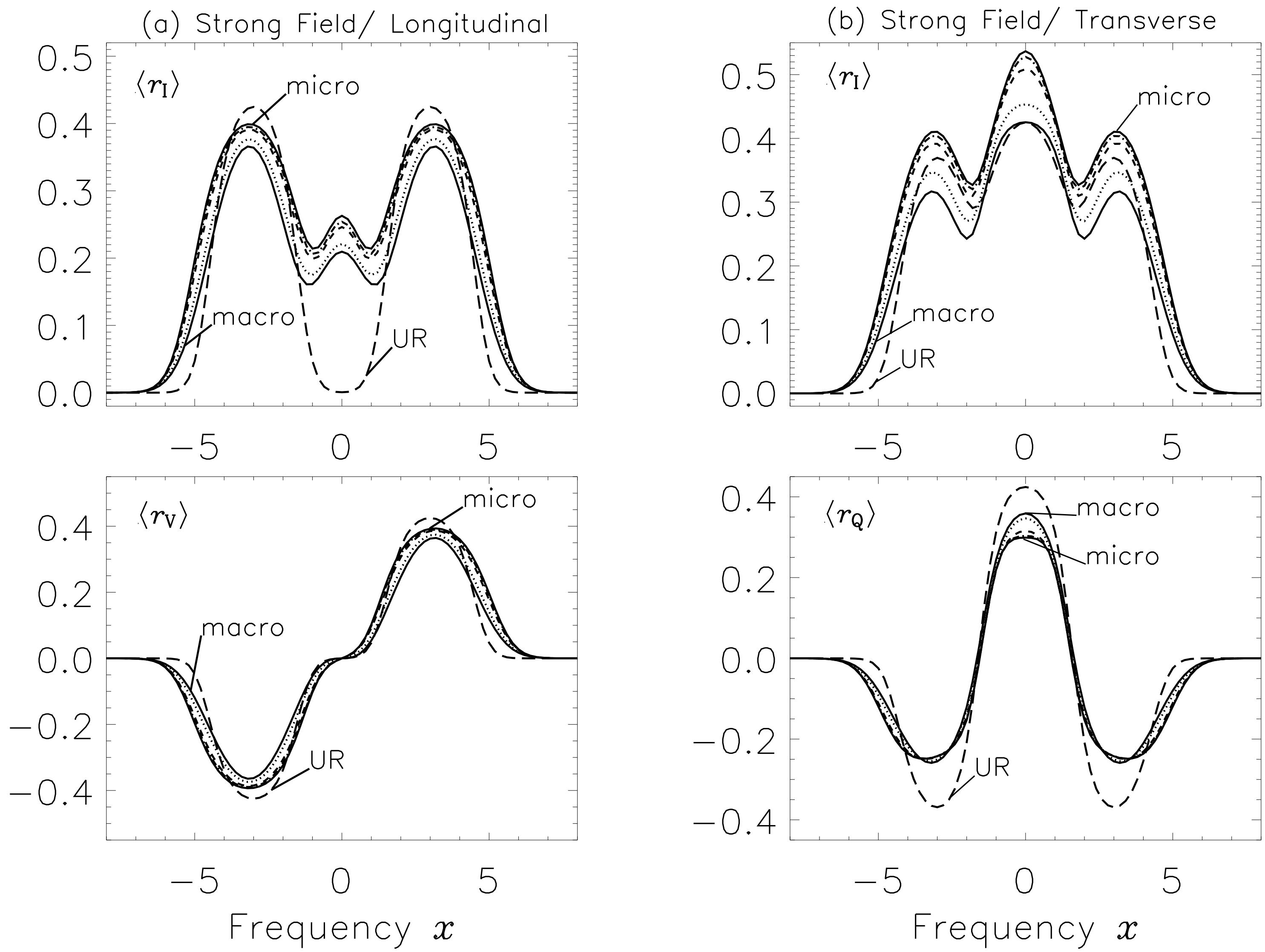
- Remarks :

$\Rightarrow$ differences between micro and macro limits significant for $\beta \simeq 10 - 100$

$\Rightarrow$ micro-turbulent limit for $\nu \geq \beta$

H. Frisch, M. Sampoorna, & K. N. Nagendra 2006, *Stochastic polarized line formation II. Zeeman line transfer in a random magnetic field*, A&A, 453, 1095-1109

Mean residual emergent Stokes parameters



x : in Doppler width units $\Delta_{\rm D} = \frac{\nu_0}{c}\sqrt{v_{\rm th}^2 + v_{\rm turb}^2}$; $\beta = \kappa_{\rm line}/\kappa_{\rm cont} = 10$; vector magnetic field distribution $P(\boldsymbol{B})$: isotropic Gaussian ; Zeeman shifts by the mean field and rms fluctuations $\Delta|\boldsymbol{B}|_0 = 3.$, $\Delta\sigma = 1/\sqrt{2}$; left panels : $\langle r_{\rm I,V}\rangle$ for $\theta_0 = 0°$; right panels $\langle r_{\rm I,Q}\rangle$ for $\theta_0 = 90°$

Conclusions

- We have developed a general method for solving the turbulent Zeeman line formation problem. It can be used to model polarized line profiles observed in the solar spectrum.

- There is a renewed interest on "photospheric general magnetic field". Recent works by Stenflo and Trujillo Bueno (NATURE : 2004, 430, 304 and 326) talk of the "hidden nature" of solar photospheric fields. They suggest that these hidden fields are certainly turbulent in nature.

- To examine their suggestion, one has to formulate and solve, Zeeman as well as Hanle effect line transfer problems with turbulent magnetic/velocity fields. Our work has direct relevance to these efforts.

- In the first part of my talk I discussed Zeeman line profiles formed under the so called LTE (local thermodynamic equilibrium).

- LTE is a good assumption in collision dominated parts of the atmosphere (photospheric layers)

- In the outermost layers of the Sun (upper photosphere and chromosphere), the densities are small. The dominant means of communication between the atoms is by scattering of photons and not by collisions.

- Therefore non-LTE is a correct description of radiation transfer in these layers.

- In the next part of my talk I discuss only about line scattering in the presence of magnetic fields.

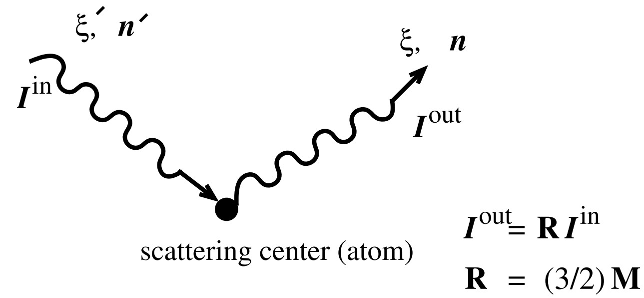
- Fields are assumed to be directed (non-turbulent).

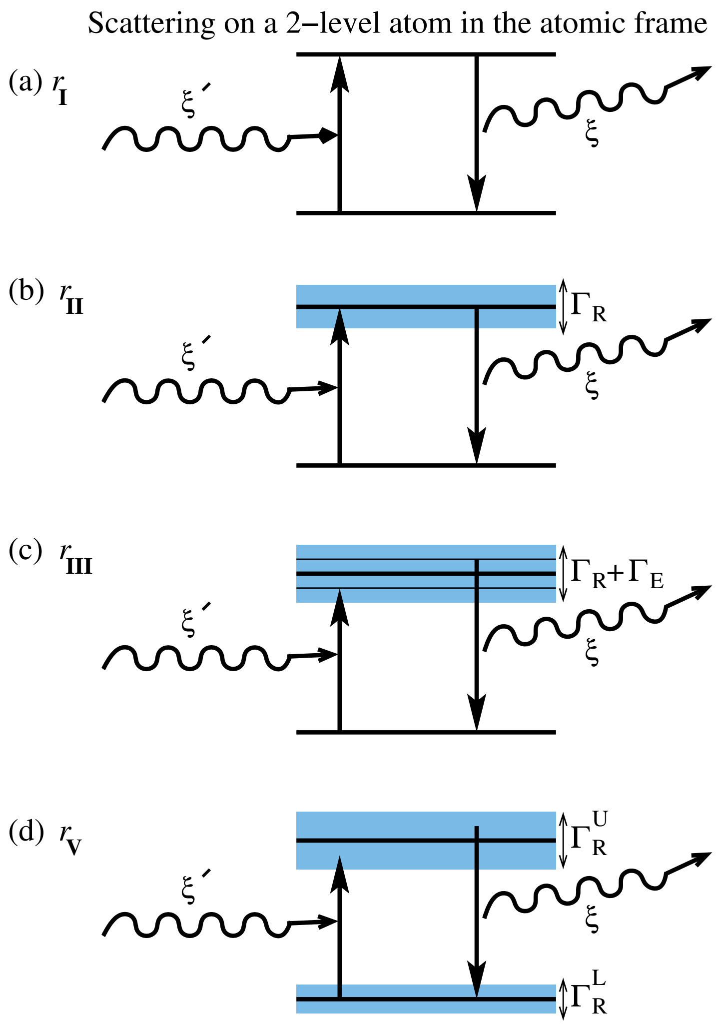
Part II

Scattering theory and Line formation

Partial Frequency Redistribution (PRD)

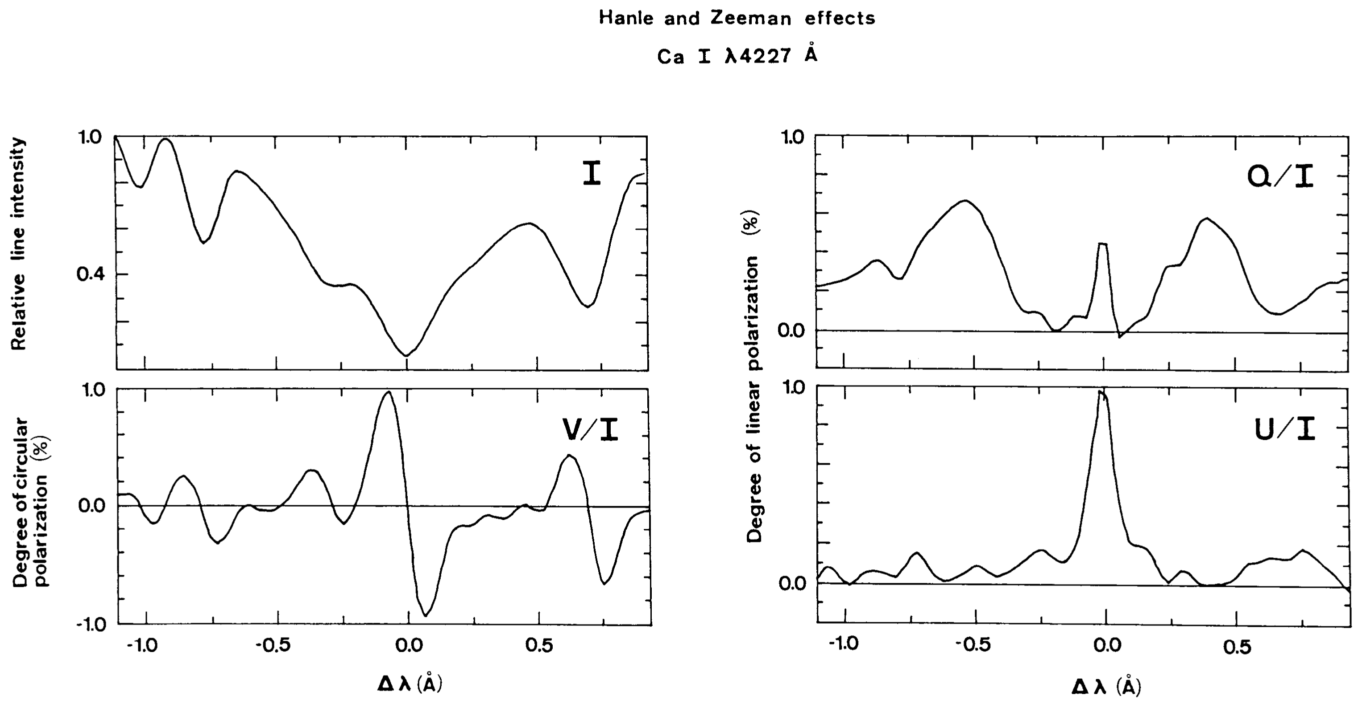
- A proper description of the angle-frequency correlation in a scattering event goes under the name of partial frequency redistribution.





- transition between two sharp levels: (unphysical) $r_{\rm I} = \delta(\xi - \xi')$.
- radiatively broadened upper level: $r_{\rm II} = \phi(\xi')\delta(\xi - \xi')$.
- radiatively + collisionally broadened upper level: $r_{\rm III} = \phi(\xi')\phi(\xi) \rightarrow$ Complete Redistribution (CRD).
- both upper and lower level broadened.
- Lab frame functions denoted R are obtained by convolving r with a Maxwellian.

Observational Motivation to develop scattering theory



Hanle and Zeeman effects in an active region 70 sec of arc inside the Solar limb. Courtesy: Stenflo (1982)

- Strong resonance lines like Ca I 4227 Å are characterized by broad damping wings surrounding a Doppler core in I, and Large Wing Polarization.
- Such lines can be modeled only when **PRD** is taken into account, especially the near wing maxima in Q/I. PRD is a better description of line scattering.
- **Reduced** amplitude of the Q/I core peak, and **appearance** of polarization peak in the core of U/I, compared to the quite-Sun case, are due to the Hanle effect.

Polarized PRD Line Scattering Theory - Recent Formulations

- To use such strong lines for magnetic field diagnostics, we need a **self-consistent scattering theory for PRD in arbitrary strength magnetic fields**.
- Such a polarized PRD matrix in the presence of arbitrary strength magnetic field, is called as **Hanle-Zeeman Redistribution Matrix** (Stenflo 1998).
- Hanle-Zeeman Regime$\Longrightarrow$ the entire field strength regime from zero field to the completely separated Zeeman components.
- Bommier (1997) has formulated a quantum electrodynamic (QED) theory of polarized PRD matrices, taking also into account the collisions.
- Bommier & Stenflo (1999) formulated a classical time-dependent oscillator theory of polarized PRD matrices, taking also into account the collisions.
- Both of them formulated the scattering theory **only in the atomic rest frame**.
- But we need **Laboratory Frame expressions**, to plug them in transfer equation.
- We have derived such Lab frame expressions in a series of papers.

Classical theory of Scattering

- In classical scattering theory the atom is assumed to be a oscillator.

- Let $\boldsymbol{E}'$ be the Electric vector of the radiation incident on this atom (it is the driving force).

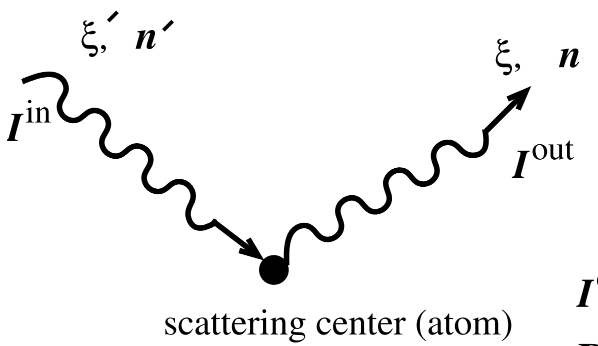
- The oscillator equation for this problem is :

$$\frac{d^2\boldsymbol{r}}{dt^2} + \frac{e}{m}(\boldsymbol{v} \times \boldsymbol{B}) + \gamma\frac{d\boldsymbol{r}}{dt} + \omega_0^2\boldsymbol{r} = -\frac{e}{m}\boldsymbol{E}'$$

- Solving this equation, we finally obtain the scattered electric vector $\boldsymbol{E}$, from which the Stokes parameters are constructed.

- The details of derivation come from standard technique of Classical Electrodynamics.

Classical Theory/ Mueller & Jones matrices



$I^{\text{out}} = \mathbf{R}\, I^{\text{in}}$

$\mathbf{R} = (3/2)\,\mathbf{M}$

- Mueller Scattering Matrix $\mathbf{M}$ gives a complete description of scattering. It is written as

$$\mathbf{M} = \mathbf{T}\,(\mathbf{w} \otimes \mathbf{w}^*)\,\mathbf{T}^{-1},$$

where $\mathbf{T}$ $\rightarrow$ Mathematical transformation matrices, and symbol $*$ denote complex conjugation.

- Jones scattering matrix $\mathbf{w}$ is given by

$$w_{\alpha\,\beta} \sim \sum_q \left[\frac{r_q(t,\,\xi')}{E'_{q,\,0}}\right] \varepsilon_q^{\alpha\,*} \varepsilon_q^{\beta}\,, \qquad q = 0, \pm 1$$

where $E'_{q,\,0}$ $\rightarrow$ amplitude of incoming monochromatic plane wave,
$\varepsilon_q^{\alpha,\,\beta}$ $\rightarrow$ geometrical factors for outgoing (α) and incoming (β) radiation,
ξ' $\rightarrow$ frequency of the incident radiation in the atomic frame,
and r_q is the solution of the oscillator equation.

Classical Theory/ Oscillator Equation Solution

- Time-dependent solution of the oscillator equation is

$$r_q(t,\, \xi') = -\mathrm{i}\phi_\gamma(\nu_0 - q\nu_\mathrm{L} - \xi') \times \begin{cases} e^{-2\pi \mathrm{i}\xi' t} & \text{stationary soln.} \\ e^{-2\pi \mathrm{i}(\nu_0 - qg\nu_\mathrm{L} - \mathrm{i}\gamma/4\pi)t} & \text{transitory soln.} \end{cases}$$

$$\text{Profile Function} \quad \phi_\gamma(\nu_0 - q\nu_\mathrm{L} - \xi') = \frac{1}{\pi \mathrm{i}} \frac{1}{\nu_0 - qg\nu_\mathrm{L} - \xi' - \mathrm{i}\gamma/4\pi}$$

ν_0, ν_L $\rightarrow$ line center and Larmor frequency; $g \rightarrow$ Landé factor, $\gamma \rightarrow$ classical damping constant.

- To determine the **spectral properties** of scattered radiation, take **Fourier transform** of the emitted wave train, $\tilde{r}_q$.

- **Truncate** the Fourier transform over the **collisions time** $\tau_c = 2/\gamma_c$, with γ_c the elastic collisional damping constant.

- Perform the **ensemble average** $\langle \tilde{r}_q \tilde{r}^*_{q'} \rangle$ to take care of randomness of the collisions.

Classical Theory/ Bilinear product

- The Bilinear Prod. $\langle \tilde{r}_q \tilde{r}^*_{q'} \rangle$ contains all the **PRD info**. In **atomic frame**

$$\langle \tilde{r}_q \tilde{r}^*_{q'} \rangle \sim \cos\beta_{q-q'} e^{\mathrm{i}\beta_{q-q'}} \Big\{ A\Phi^{\gamma+\gamma_c}_{qq'}(\xi')\delta(\xi-\xi') \quad \text{freq. coh. scat. (stationary)}$$

$$+B\cos\alpha_{q-q'} e^{\mathrm{i}\alpha_{q-q'}} \Phi^{\gamma+\gamma_c}_{qq'}(\xi')\Phi^{\gamma+\gamma_c}_{qq'}(\xi) \Big\}, \quad \text{non–coh. scat. (transitory)}$$

generalized profile func. $$\Phi^{\gamma+\gamma_c}_{qq'} = \frac{1}{2}\left[\phi_{\gamma+\gamma_c}(\nu_0 - q\nu_{\mathrm{L}} - \xi) + \phi^*_{\gamma+\gamma_c}(\nu_0 - q'\nu_{\mathrm{L}} - \xi)\right]$$

$$\text{Hanle angles}: \quad \tan\beta_{q-q'} = \frac{(q-q')2\pi\nu_{\mathrm{L}}}{\gamma+\gamma_c}, \quad \tan\alpha_{q-q'} = \frac{(q-q')2\pi\nu_{\mathrm{L}}}{\gamma+\gamma_c/2}.$$

A and B are Branching ratios "**imported from quantum theory**".

- Doppler convolution is necessary to go from Atomic $\rightarrow$ Laboratory frame.
- After transformation $\langle \tilde{r}_q \tilde{r}^*_{q'} \rangle$ consists of "**Magnetic Redistribution Functions**" in Lab frame.

Classical Theory/ Magnetic Redistribution functions

- The type II Magnetic Redistribution functions are given by:

$$R_{\rm II}^{q}(x,x',\Theta) = \frac{1}{\pi \sin\Theta} \exp\left\{-\left[\frac{x-x'}{2\sin(\Theta/2)}\right]^2\right\} \mathcal{H}\left(\frac{x+x'-2qv_B}{2\cos(\Theta/2)}, \frac{a}{\cos(\Theta/2)}\right).$$

- Type III Magnetic Redistribution functions are given by

$$R_{\rm III}^{qq'}(x,x',\Theta) = \frac{1}{\pi^2 \sin\Theta} \int_{-\infty}^{+\infty} du e^{-u^2} \left[\frac{1}{a+{\rm i}(v'_q-u)}\right] \mathcal{H}\left(\frac{v_{q'}}{\sin\Theta} - u\cot\Theta, \frac{a}{\sin\Theta}\right).$$

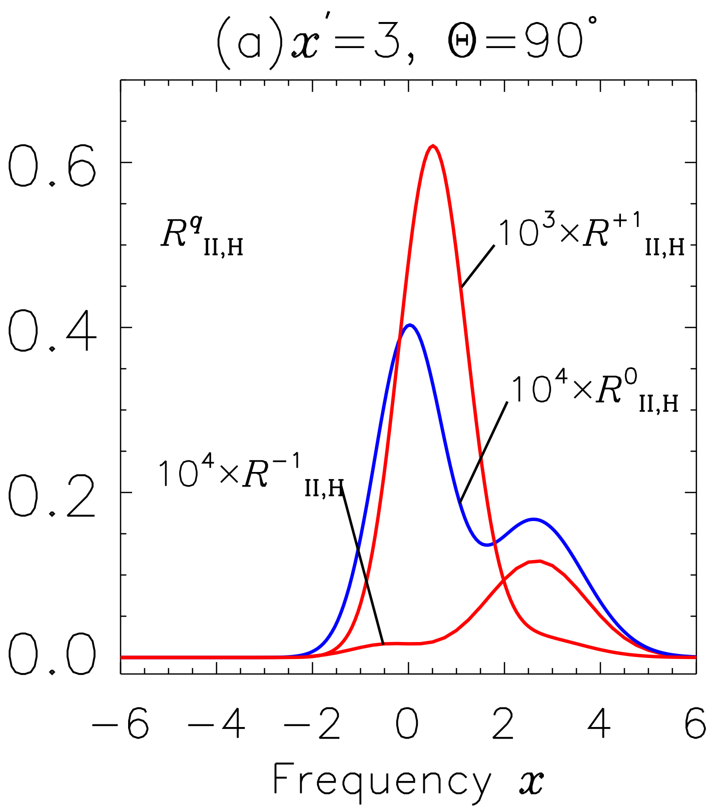
- $q = q' = 0$ correspond to **scalar** redistribution function of Hummer (1962).

- We have introduced in the above equations

$$x = \frac{\nu_0 - \nu}{\Delta\nu_{\rm D}}; \quad v_q = x - qv_B; \quad v_B = \frac{g\nu_{\rm L}}{\Delta\nu_{\rm D}}; \quad a = \frac{\gamma+\gamma_c}{4\pi\Delta\nu_{\rm D}},$$

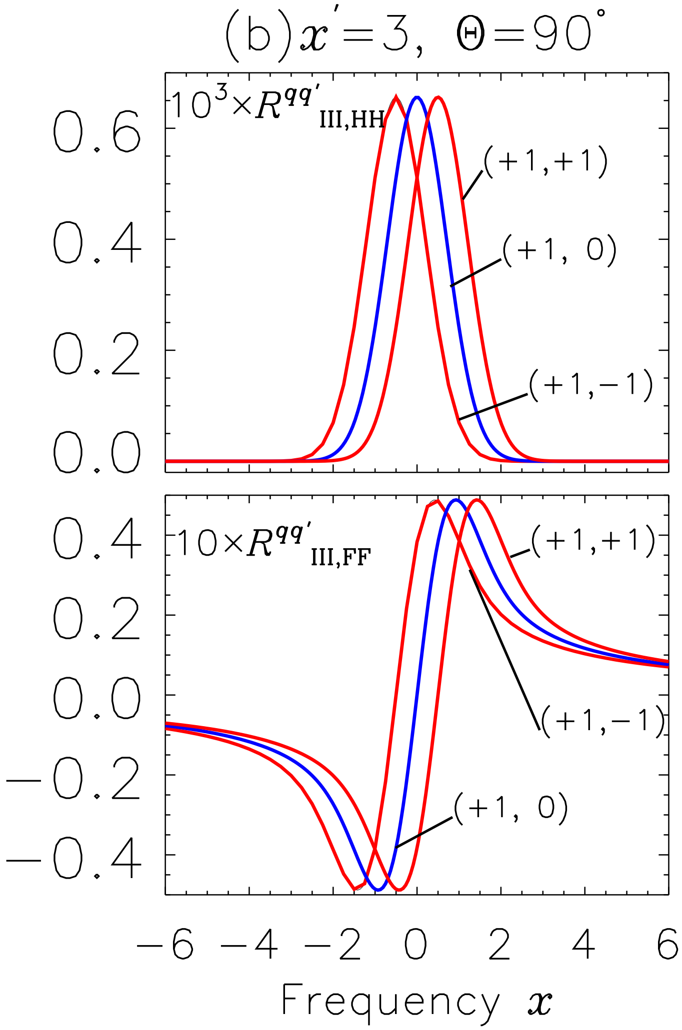
$$\text{Complex Voigt Func.} \quad \mathcal{H}(x,a) = H(x,a) - {\rm i}F(x,a),$$

Type II Magnetic Redistribution Functions



- $a=0.001$, $v_B=g\nu_{\mathrm{L}}/\Delta\nu_{\mathrm{D}}=0.5$
- $R^0_{\mathrm{II,H}}$ → double peaked behavior (blue line)
- For $q=\pm1$ → 2 competing processes are there:
  → Freq. coherence (peak $x\sim x'$)
  → Mag. coherence (peak at $x\sim qv_B$)
- $R^{-1}_{\mathrm{II,H}}$ → Freq. coherence dominates (red line)
- $R^{+1}_{\mathrm{II,H}}$ → Mag. coherence dominates (red line)

Type III Magnetic Redistribution Functions



- For a 90° scattering,

  $$R^{qq'}_{\mathrm{III,XY}} = X(x' - qv_B, a)\, Y(x - q'v_B, a),$$

  X and Y stand for normalized Voigt (H) and/or Faraday-Voigt (F) functions
- Peak position and shape is determined by $Y(x - q'v_B, a)$
- Amplitude is determined by $X(x' - qv_B, a)$
- Notice the expected CRD like behavior of RFs.

M. Sampoorna, K. N. Nagendra, & J. O. Stenflo 2007a, *Hanle-Zeeman redistribution matrix I. Classical theory expressions in the laboratory frame*, ApJ, 663, 625. Here we develop the classical theory framework for Hanle-Zeeman Effect.

Comparison with QED theory of Bommier (1997)

- To check the correctness of the classical theory expressions, we compared with the corresponding QED theory expressions, for the normal Zeeman triplet.

- Classical and QED theory give exactly the same expression for **R**, if we make the following identification

$$\gamma = \Gamma_{\rm R} + \Gamma_{\rm I}, \quad \gamma_c = \Gamma_{\rm E}, \quad D^{(K)} = \gamma_c/2 \;\; \text{for} \;\; K = 1,\, 2$$

$\Gamma_{\rm R} \rightarrow$radiative rate;
$\Gamma_{\rm I} \rightarrow$inelastic collision rate;
$\Gamma_{\rm E} \rightarrow$elastic collision rate;
$D^{(K)} \rightarrow$depolarizing collisions (note $D^{(0)} = 0$).

M. Sampoorna, K. N. Nagendra, & J. O. Stenflo 2007b, *Hanle-Zeeman Redistribution Matrix II. Comparison of Classical and Quantum Electrodynamic Approaches*, ApJ, 670, 1485-1504. In this paper, the Equivalence of classical and Quantum PRD expressions are proved.

Hanle-Zeeman Line Formation Problem with PRD

- The vector radiative transfer equation for the Stokes vector $\boldsymbol{I} = (I, Q, U, V)^{\mathrm{T}}$ is

$$\mu \frac{\partial}{\partial \tau} \boldsymbol{I}(\tau, x, \boldsymbol{n}) = [\boldsymbol{\Phi} + r\mathbf{E}]\boldsymbol{I}(\tau, x, \boldsymbol{n}) - \{(r\mathbf{E} + \epsilon \boldsymbol{\Phi}) B_{\nu_0} \boldsymbol{U} + \boldsymbol{S}_{\mathrm{scat}}(\tau, x, \boldsymbol{n})\},$$

$$\boldsymbol{S}_{\mathrm{scat}}(\tau, x, \boldsymbol{n}) = \int \frac{d\boldsymbol{n}'}{4\pi} \int dx' \mathbf{R}(x, \boldsymbol{n}; x', \boldsymbol{n}'; \boldsymbol{B}) \boldsymbol{I}(\tau, x', \boldsymbol{n}').$$

$\tau \rightarrow$ line-center optical depth;

$B_{\nu_0} \rightarrow$ Planck function; $\boldsymbol{U} = (1\,0\,0\,0)^{\mathrm{T}}$,

$\boldsymbol{n}(\vartheta, \varphi) \rightarrow$ propagation direction of the ray (ϑ, φ defined w.r.t. the slab normal); $\mu = \cos\vartheta$;

$\boldsymbol{\Phi} \rightarrow$ 4×4 Zeeman line absorption matrix;

$\mathbf{E} \rightarrow$ 4×4 unity matrix;

$r \rightarrow$ ratio of continuum to line center opacity;

$\epsilon \rightarrow$ photon destruction probability per scattering

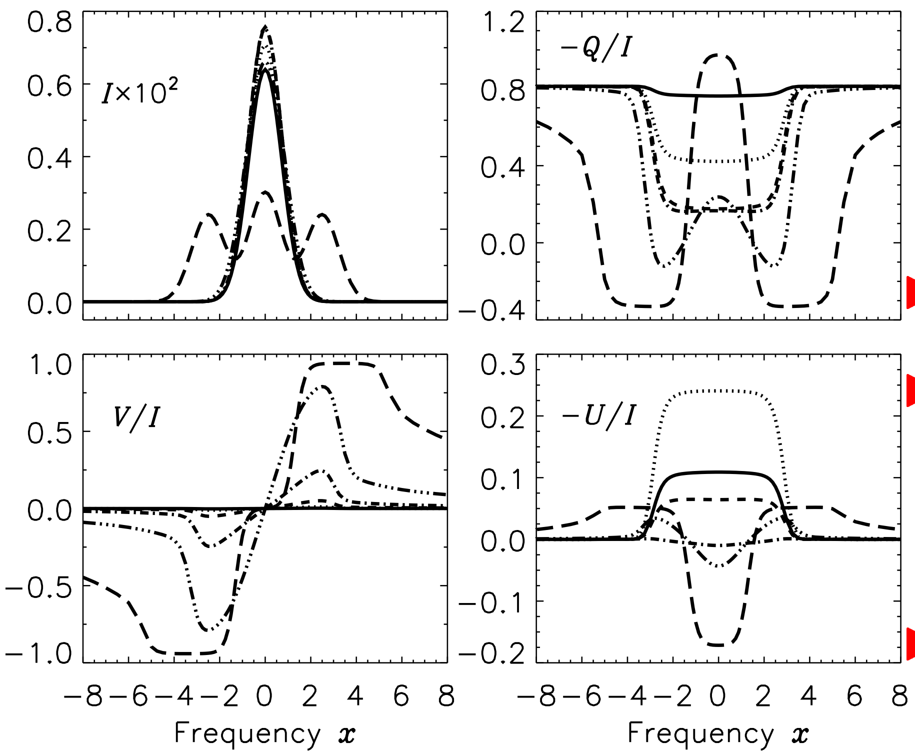
A Perturbation method for Hanle-Zeeman PRD theory

- We solve Hanle-Zeeman line transfer problem with Angle dependent PRD, using a perturbation approach.
- Because, there does NOT exist any other numerical method to solve this complex problem.
- The perturbation method employs 2 stages

  ⇒ In Stage 1 - we solve the Hanle-Zeeman scattering problem neglecting Zeeman absorption.

  ⇒ In Stage 2 - the intensity solution from stage 1 is used as a input and the full problem of Zeeman absorption + Hanle-Zeeman scattering is solved iteratively.
- Iteration is continued until a "convergence is reached".
- In the past, the strong field Zeeman absorption and weak field Hanle scattering radiative transfer equation were solved independently - and NOT together. This calculation is the FIRST attempt to solve the combined problem.

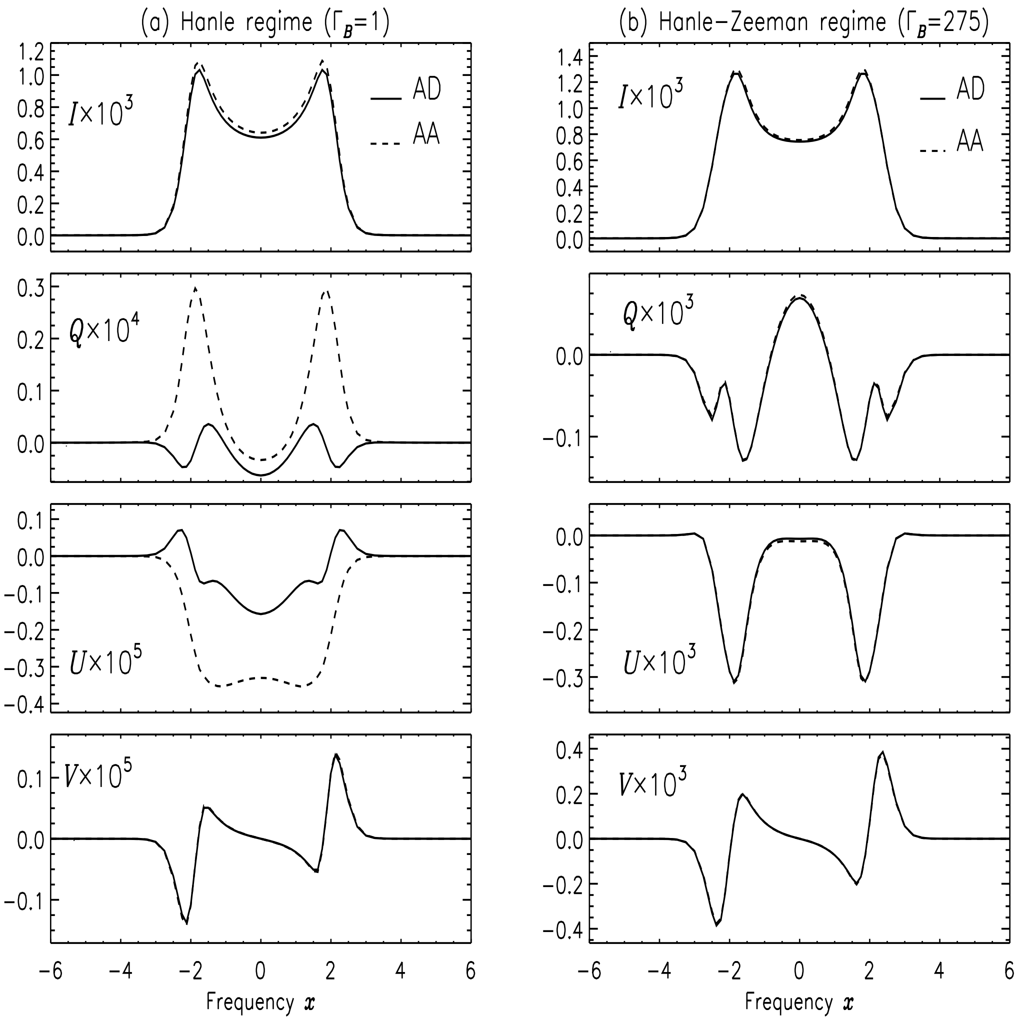
Some Results from Hanle-Zeeman PRD theory - I



- A nearly 90° scattering of the unpolarized incident ray (at μ'=0.95, φ'=0°), by a very thin (T=0.02) slab medium (almost like single scattering).
- Purely scattering medium, with $R_{\rm II}$.
- Canopy-like field $(\vartheta_B, \varphi_B) = (90°, 45°)$. Field splitting v_B varied in steps of 5 from 0.0008 to 2.5.
- Emergent Soln. for μ=0.11 and φ=0°.

- Notice a Smooth Transition from Hanle regime for weak fields to Zeeman regime for strong fields, as the field strength is increased.
- The Stokes V is Generated purely due to Hanle-Zeeman scattering in this case.

Some Results from Hanle-Zeeman PRD theory - II



- Emergent solution for μ=0.11.
- Isothermal, Self-emitting thin ($T = 20$) slab.
- $R_{\rm II}$ dominating ($\Gamma_{\rm E}/\Gamma_{\rm R}=0.1$)
- (ϑ_B, φ_B)=$(30°, 45°)$.
- (a, ϵ, r)=$(10^{-3}, 10^{-4}, 10^{-9})$.
- Clearly, for very strong fields there is no significant difference between the AA and AD profiles.

- The Hanle Γ_B parameter is defined as $\Gamma_B = geB/(2m_e c\Gamma_{\rm R})$, in standard notations.

Conclusions

- In the line formation theories we need to account for partial frequency redistribution (PRD).
- The relevant PRD functions in arbitrary fields (Hanle-Zeeman effect) are now derived in our series of papers.
- From the classical approach we derived analytic forms of the Hanle-Zeeman redistribution matrix in the Laboratory frame, for use in radiative transfer theory (Sampoorna et al. 2007a, ApJ).
- For a $J = 0 \rightarrow 1 \rightarrow 0$ triplet transition, we show the 'equivalence' between the classical and QED theories (Sampoorna et al. 2007b, ApJ).
- We incorporated the Hanle-Zeeman redistribution matrix into the transfer equation and it is solved using a perturbative approach, which is the first ever solution of this problem (Sampoorna et al. 2007c, ApJ, Submitted).

Thank You for your attention

List of Publications

PART-I

- Frisch, H., Sampoorna, M., & Nagendra, K. N. 2005, A&A, 442, 11-28
- Frisch, H., Sampoorna, M., & Nagendra, K. N. 2006, A&A, 453, 1095-1109
- Frisch, H., Sampoorna, M., & Nagendra, K. N. 2006, in ASP Conf. Ser. 358, Solar Polarization 4, ed. R. Casini, & B. W. Lites, 126-131
- Sampoorna, M., Nagendra, K. N., & Frisch, H. 2007, JQSRT, 104, 71-85
- Frisch, H., Sampoorna, M., & Nagendra, K. N. 2007, Mem. S. A. It., 78, 142-147
- Sampoorna, M., Frisch, H., & Nagendra, K. N. 2007, New Astronomy, In Press
- Sampoorna, M., Nagendra, K. N., Frisch, H., & Stenflo, J. O. 2007, A&A, (Submitted)

List of Publications

PART-II

- Sampoorna, M., Nagendra, K. N., & Stenflo, J. O. 2007, ApJ, 663, 625-642
- Sampoorna, M., Nagendra, K. N., & Stenflo, J. O. 2007, ApJ, 670, 1485-1504
- Sampoorna, M., Nagendra, K. N., & Stenflo, J. O. 2007, ApJ, Submitted
- Sampoorna, M., Nagendra, K. N., & Stenflo, J. O. 2007, in ASP Conf. Ser. ..., Solar Polarization 5, ed. S. V. Berdyugina, K. N. Nagendra, & R. Ramelli, Submitted
- Nagendra, K. N., & Sampoorna, M. 2007, in ASP Conf. Ser. ..., Solar Polarization 5, ed. S. V. Berdyugina, K. N. Nagendra, & R. Ramelli, Submitted
- Sampoorna, M., Nagendra, K. N., & Frisch, H. 2007, A&A, Submitted

PART-III

- Yee Yee Oo, Sampoorna, M., Nagendra, K. N., Sharath Ananthamurthy, & Ramachandran, G. 2007, JQSRT, 108, 161-179
- Yee Yee Oo, Phyu Phyu San, Sampoorna, M., Nagendra, K. N., & Ramachandran, G. 2007, in ASP Conf. Ser. ..., Solar Polarization 5, ed. S. V. Berdyugina, K. N. Nagendra, & R. Ramelli (Submitted)