

Astronomical Techniques

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Notes: <http://www.iiap.res.in/people/personnel/ereddy/ereddy.html>

Reference:

W.M. Smart: Text book on Spherical Astronomy

R.M. Green: Spherical Astronomy

Kaitchuk, R.H: Astronomical Photometry

Erika Bohm-Vitense: Basic stellar observations- Voll

Assignment

- 1) Let A and B represent two places. Place A is at $24^{\circ} 18' \text{ N}$ and place B is at $36^{\circ} 47' \text{ N}$. They have Longitudes $133^{\circ} 39' \text{ E}$ and $125^{\circ} 24' \text{ W}$, respectively.
 - (a) Find the great circle arc AB?
 - (b) Find the angle PAB? where P is the north pole
- 2.) What is the shortest distance between New Delhi and New York ?
(use latitudes and longitudes and distance in kms)
- 3.) How is the RA and Dec coordinate system more useful compared to other coordinate systems like: altitude-Azimuth and HA and Dec.?
- 4.) Find out best observing seasons for the star 'Vega' and the cluster Hyades? Also, find out dates on which these are overhead for an observer at the Vainu Bappu Observatory, Kavalur?

Assignment

- 5.) A star whose luminosity is $200L(\text{sun})$ has an apparent bolometric magnitude $m_{\text{bol}} = 9.8$. Given that the Sun's absolute bolometric magnitude $M_{\text{bol}} = 4.8$, determine the distance to the star ?
- 6.) What is the radius of star of the same T_{eff} as the Sun but with luminosity 10^4 times larger?
- 7.) Determine the value of airmass for a star with RA: 10h:30m:15s and Dec: 30deg: 15':15". Observations are from IAO, Hanle on 21 Feb 2007 at 8:00PM (IST).
- 8.) For a star in certain direction the colour excess $E(B-V)$ is found out to be 1.0. Given the measured magnitudes of $V_{\text{mag}} = 8.0$ and $B_{\text{mag}} = 10.5$, find out star's unreddened color $B-V$ and its appropriate spectral class?
- 9.) Find out the star Polaris altitude from IAO, Hanle ?
- 10.) Two stars in a binary system are separated by $0.01''$ (arc-sec). Discuss the observational strategy to study them?

Assignment

- 11) The $H\gamma$ line in the spectrum of certain quasar found to to be shifted to 9000\AA . Given the best estimated H_0 (Hubble constant) determine distance to the quasar?
- 12) Images of two stars are 100 pixels apart on CCD chip of pixel size= 15micron . Given the VBT prime focus, find out the angular separation of the two stars in the sky?
- 13) Two images of a cluster taken 10 years apart showed that the cluster moved $10.5'$ (arc-minute) right angles to the line of sight. Given the wavelength shift of 0.5\AA of $H\alpha$ line, find the distance to the cluster?
- 14) A star of certain brightness is observed with HCT for 30min. exposure time. The resultant image has $S/N=60$. How large aperture is need to attain $S/N=200$ with the same set-up?
- 15.) When does the Sun reaches closest to the Polaris and what is the angle between them?

Assignment

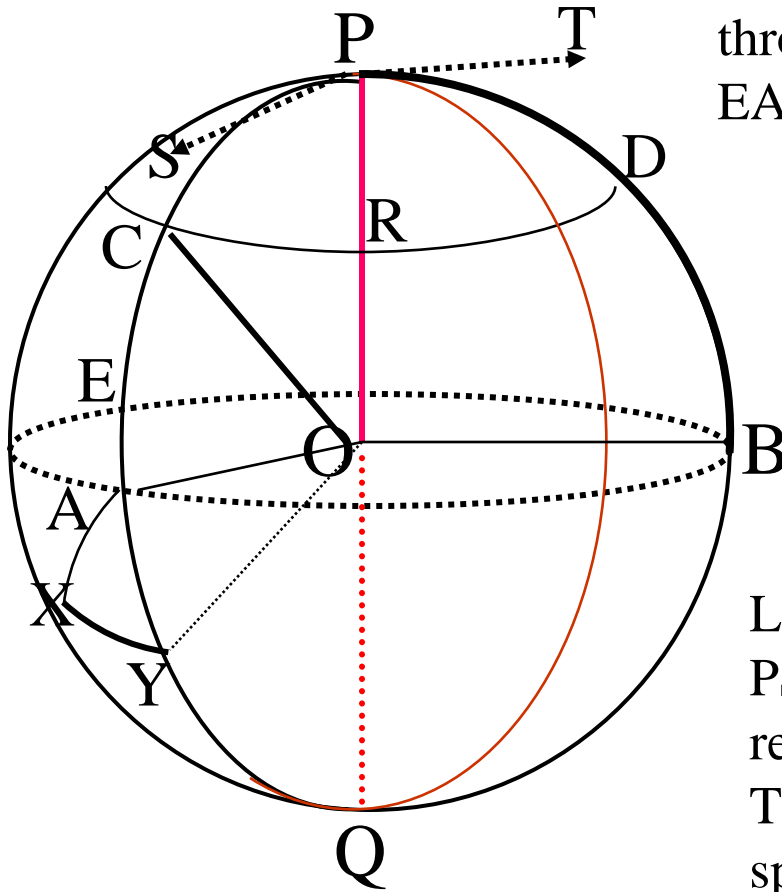
- 16) Stromgren photometry for two stars is given: star1 ($v=10.0$, $b=9.5$ and $y=8.0$) and star2 ($v=12.0$, $b=10.5$ and $y=9.0$). Discuss their relative metallicities?
- 17.) What is the field of view for the VBT telescope with the Cassegrain focus?
It is fitted with an eye piece of FOV 40° and focal length of 500cm.
- 18.) For a telescope of 50 cm the limiting magnitude (one can see thru the telescope) is 15. To see stars as faint as 26^{th} magnitude, what sort of telescope aperture one should have?
- 19.) The pole star or Polaris has 2000 epoch coordinates RA: 02:31:49.08; DEC: +89:15:50.8. Find out its coordinates for 10000 AD epoch?
- 20.) Find out plate scales and angular resolutions for HCT and VBT telescopes?

21. Evaluate the distance to the cluster using the data available below ?

<i>No</i>	<i>V</i>	<i>V-R</i>	<i>R-I</i>
<i>01</i>	<i>10.540</i>	<i>0.710</i>	<i>0.680</i>
<i>03</i>	<i>14.300</i>	<i>1.160</i>	<i>1.290</i>
<i>03</i>	<i>14.250</i>	<i>1.120</i>	<i>1.340</i>
<i>04</i>	<i>12.730</i>	<i>0.930</i>	<i>0.990</i>
<i>04</i>	<i>12.710</i>	<i>0.910</i>	<i>1.030</i>
<i>05</i>	<i>15.170</i>	<i>1.290</i>	<i>1.510</i>
<i>06</i>	<i>13.980</i>	<i>1.000</i>	<i>1.350</i>
<i>07</i>	<i>10.760</i>	<i>0.670</i>	<i>0.690</i>
<i>09</i>	<i>14.750</i>	<i>1.360</i>	<i>1.510</i>
<i>10</i>	<i>12.510</i>	<i>0.860</i>	<i>0.910</i>
<i>11</i>	<i>11.110</i>	<i>0.660</i>	<i>0.700</i>
<i>12</i>	<i>13.110</i>	<i>1.070</i>	<i>1.270</i>
<i>13</i>	<i>7.270</i>	<i>0.060</i>	<i>0.090</i>
<i>14</i>	<i>17.070</i>	<i>1.520</i>	<i>1.730</i>
<i>15</i>	<i>13.970</i>	<i>0.990</i>	<i>1.210</i>
<i>17</i>	<i>16.820</i>	<i>1.610</i>	<i>1.930</i>
<i>18</i>	<i>17.660</i>	<i>1.720</i>	<i>2.040</i>

The spherical triangle

Any plane passing through the center of a sphere cuts the surface in a circle called a **great circle**. Any other plane cutting the sphere but not passing through the center is known as a **small circle**. Here, EAB is a great circle and CD is a small circle.

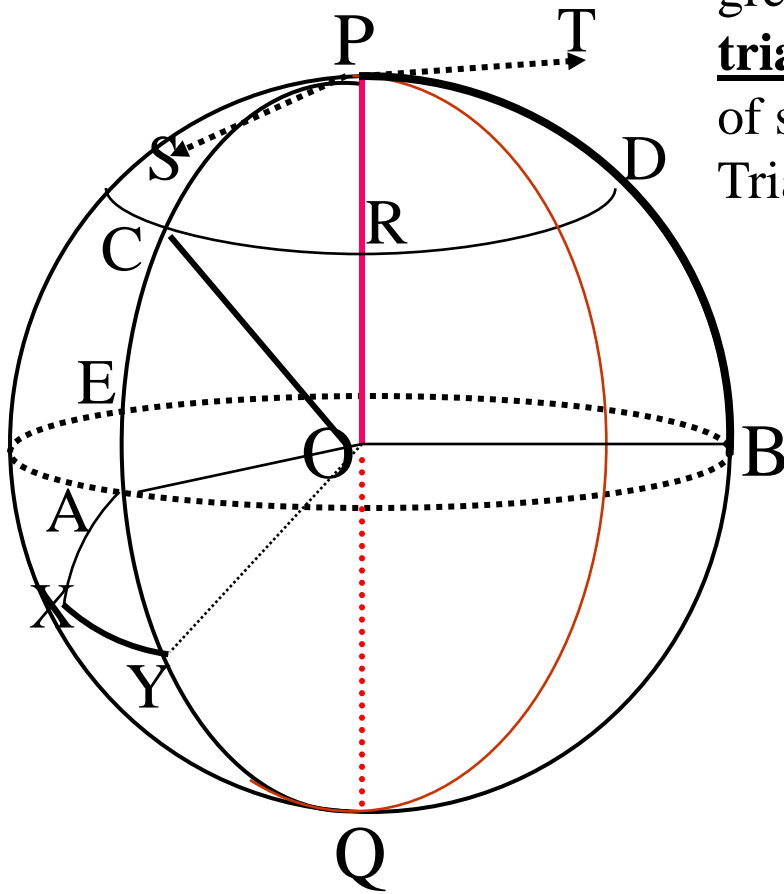


QOP is the diameter of the sphere which is \perp to the plane EAB. Plane CD is \parallel to plane EAB. OP is \perp to planes EAB and CD. **P and Q are the poles of great circle.**

Let PCAQ and PDB any other two great circles. PS and PT are the tangents to PA and PB (easy to refer parts of great circles). The angle SPT or The angle AOB (or the arc AB) is said to be a spherical angle. **A spherical angle is defined only with reference to two intersecting great circles.**

The spherical triangle

Let A, X, Y are three points lying in the same hemisphere and joined by arcs (AX, AY and XY) of great circles. **The angle AXY is called spherical triangle** and the angles at A, X, and Y are the angles of spherical triangle. Angle PCD is not a spherical Triangle as CD is not a part of great circle.



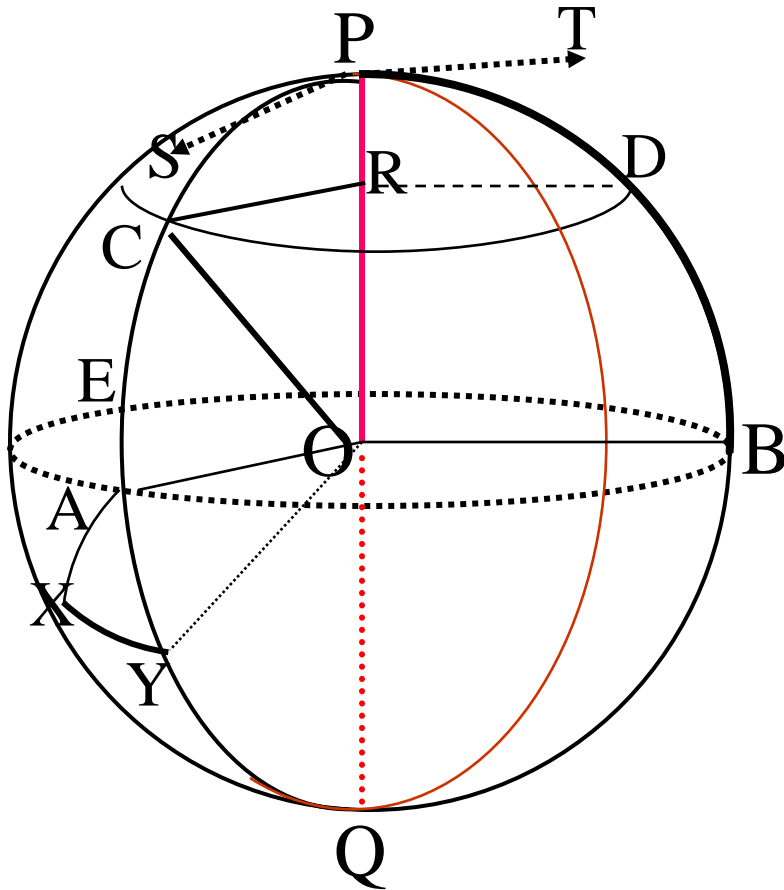
Length of great circle arc

$$\underline{\underline{AY = r \times \text{angle AOY}}},$$

Where r is the radius of the great circle, and is constant for all the great circles on the sphere and can be set to unity. Thus, arc AY is said to be the angle AOY. This angle is measured in radians.

The spherical triangle

Length of a small circle arc:



$$CD = RC \times \text{angle } CRD$$

$$AB = OA \times \text{angle } AOB$$

Since $CD \parallel EAB$, $CRD = AOB$ and $OA = RC$

Hence, $CD = (RC/OA) \times AB$

$$= (RC/OC) \times AB$$

($OC=OA$ radii of the sphere)

As $RC \perp RO$, $\cos(RCO) = RC/OC$

and as $RC \parallel OA$, $\text{angle } RCO = \text{angle } AOC$

Therefore,

$$CD = \cos RCO \times AB = AB \times \cos(AOC)$$

Thus, **$CD = AB \times \cos(AC)$**

or, since $PA = 90^\circ$

$$\mathbf{CD = AB \times \sin(PC)}$$

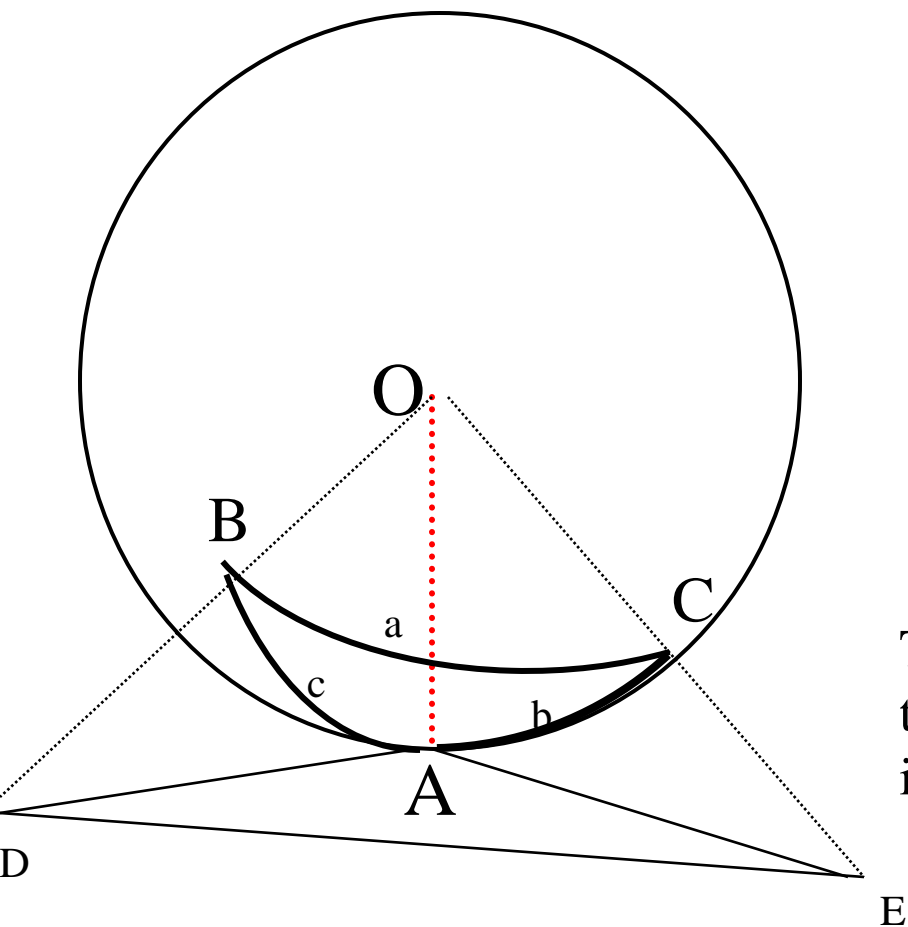
The spherical triangle: The Cosine-formula

Let ABC be a spherical triangle. Thus,

BC (arc) = angle $BOC = a$

AC (arc) = angle $AOC = b$

AB (arc) = angle $AOB = c$



AD and AE be the tangents to the great circles AC and AB respectively.

radius $OA \perp AD$ and AE

The spherical angle BAC is the angle between the tangents at A to the great circles and which is equal angle DAE .

Let angle $BAC = DAE = A$

The spherical triangle: The Cosine-formula

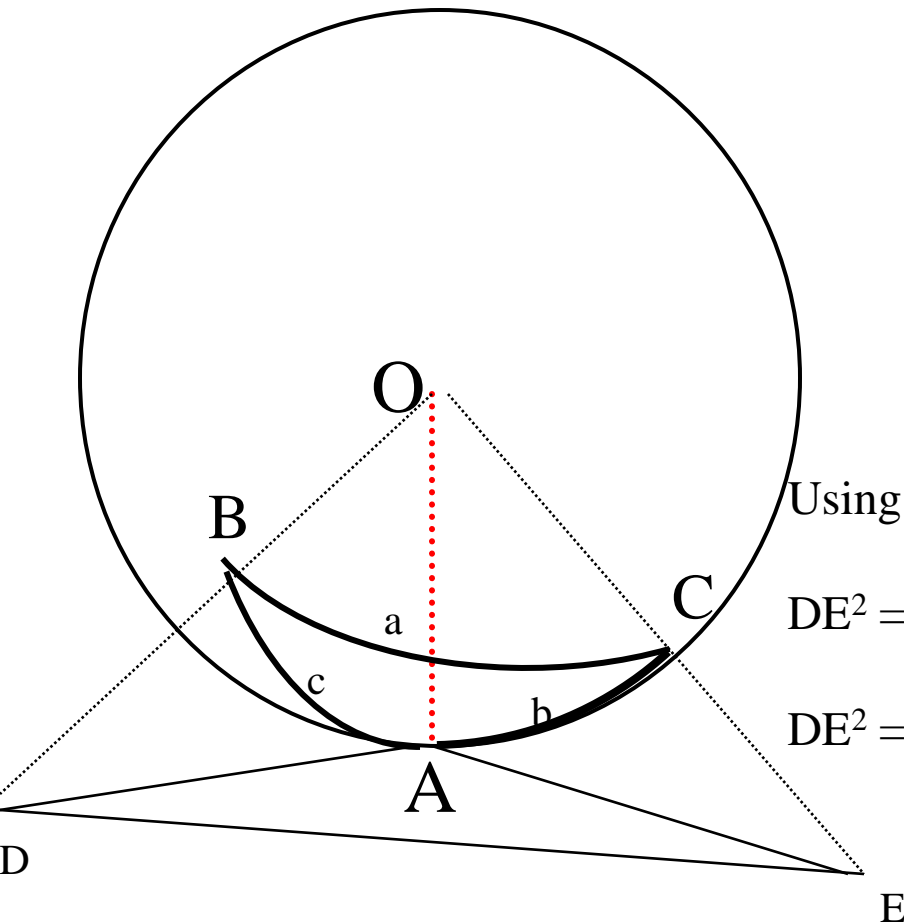
In the plane OAD, angle OAD = 90°
and angle AOD = angle AOB = c

In the triangle OAD:

$$AD = OA \tan c; \quad OD = OA \sec c \dots\dots 1$$

In the triangle OAE:

$$AE = OA \tan b; \quad OE = OA \sec b \dots\dots 2$$



Using the triangle DAE:

$$DE^2 = AD^2 + AE^2 - 2AD \cdot AE \cos DAE$$

$$DE^2 = OA^2 \tan^2 c + OA^2 \tan^2 b - 2(OA \tan c \cdot OA \tan b \cos A) \dots\dots 3$$

The spherical triangle: The Cosine-formula

$$DE^2 = OA^2 \tan^2 c + OA^2 \tan^2 b - 2(OA \tan c \cdot OA \tan b \cos A) \dots 3$$

$$DE^2 = OA^2 (\tan^2 c + \tan^2 b - 2 \tan b \cdot \tan c \cos A) \dots \dots \dots 4$$

Similarly from triangle DOE

$$DE^2 = OD^2 + OE^2 - 2OD \cdot OE \cos DOE$$

$$= OA^2 \sec^2 c + OA^2 \sec^2 b - 2OA \sec c \cdot OA \sec b \cdot \cos A$$

or

$$DE^2 = OA^2 (\sec^2 c + \sec^2 b - 2 \sec b \cdot \sec c \cdot \cos A) \dots \dots \dots 5$$

From (4) and (5)

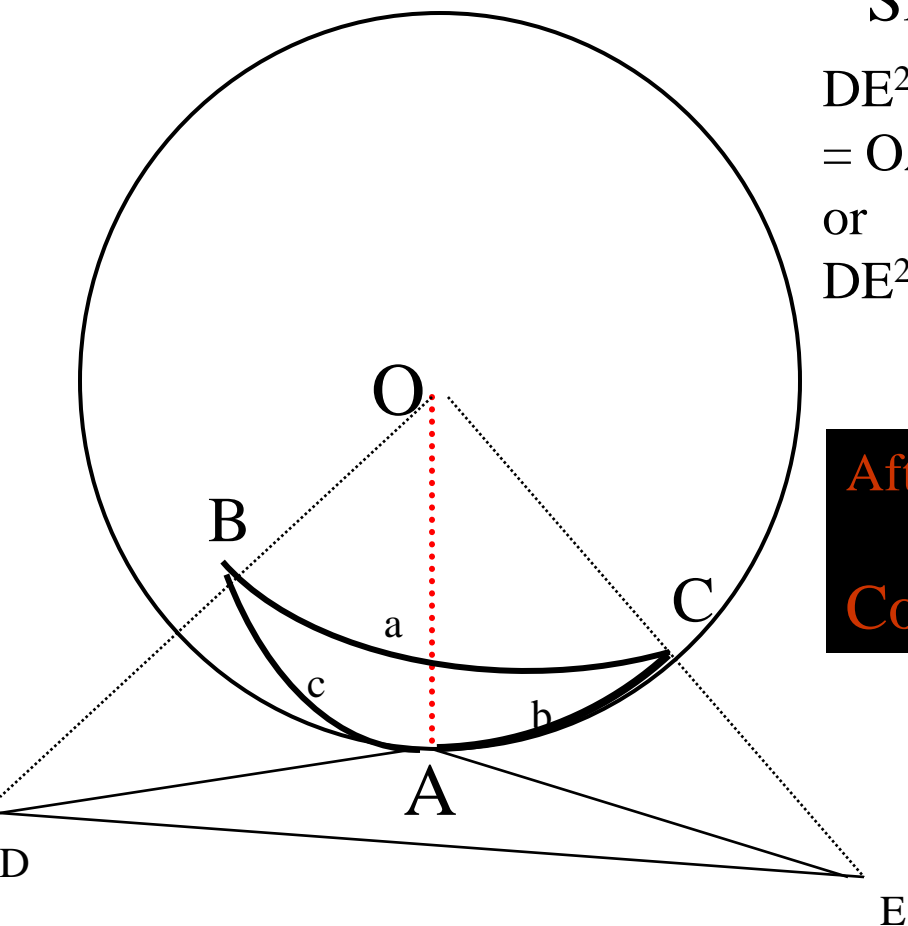
After simplification

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

This is called cosine formula

Similarly sine formula

$$\sin A / \sin a = \sin B / \sin b = \sin C / \sin c$$



The earth: longitude and latitude

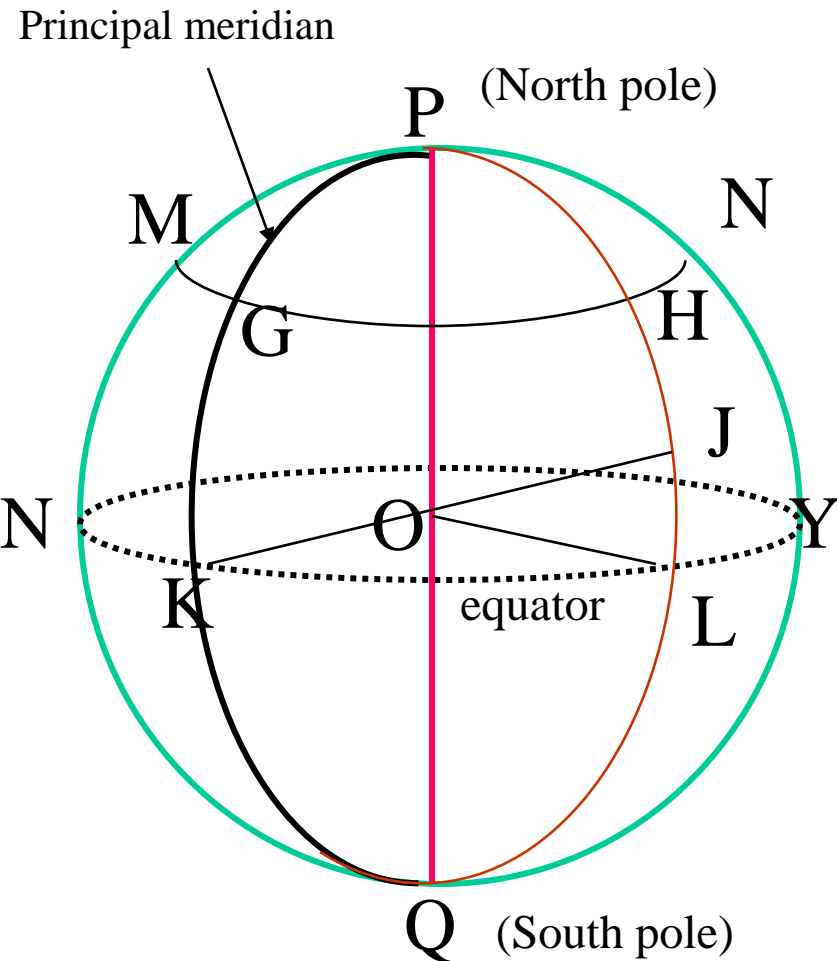
The great circle (NKLY) perpendicular to the earth's spin axis (POQ) is called the equator.

PGK is the meridian which passes through the Greenwich Observatory is called principal meridian or GM.

PHLQ is any other meridian intersecting the equator at L.

Longitude:

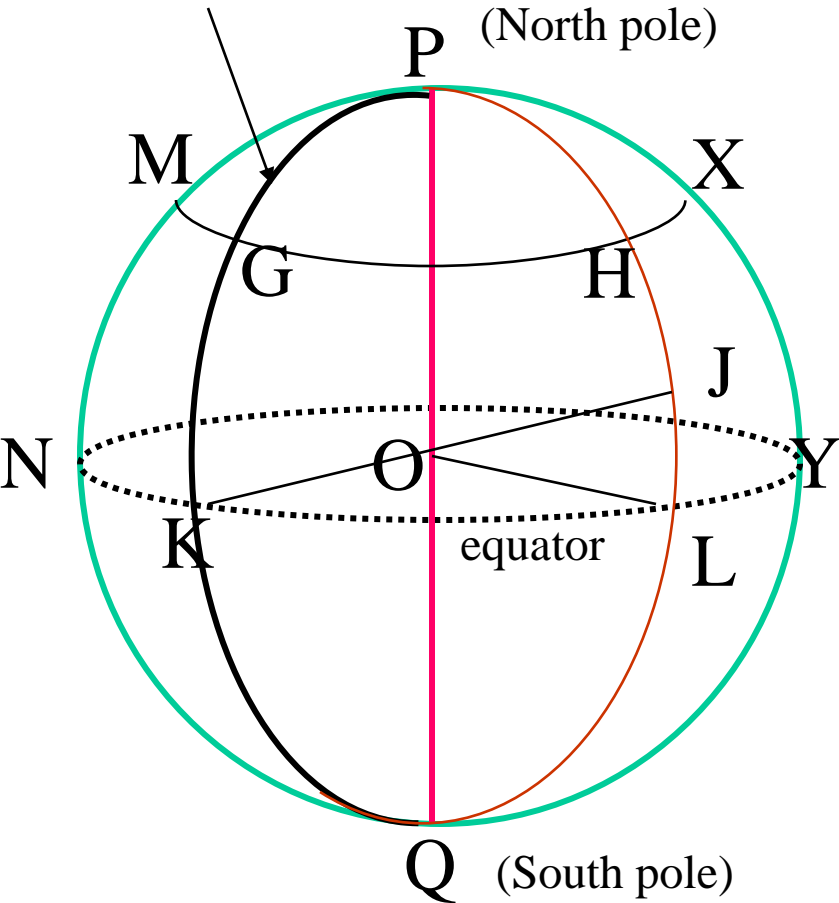
Angle KOL or the arc KL on the equator is defined to be the longitude of the meridian PHLQ. Measured from 0-180deg east of GM and 0-180deg west of GM. All the positions on meridian PHLQ have same longitude



The earth: longitude and latitude

Latitude:

Principal meridian



Let 'J' be a place on the meridian PHJQ. The angle LOJ or the great circle arc LJ is the latitude of the place J. If the place is above the equator it is said to be latitude north, if it is below it is latitude south.

Thus position of any place on the Earth is specified by the two fundamental great circles: the equator and the principal meridian.

If \emptyset be the latitude of J. $POL=90^\circ$. Therefore, $PJ = 90^\circ - \emptyset$ is called **co-latitude of J**. All places which have the same latitude as Greenwich lie on the small circle MGHX, which is parallel to the equator, called **parallel of latitude**.

The earth: longitude and latitude

Length of the small circle HX is given in terms of the length of the equator as

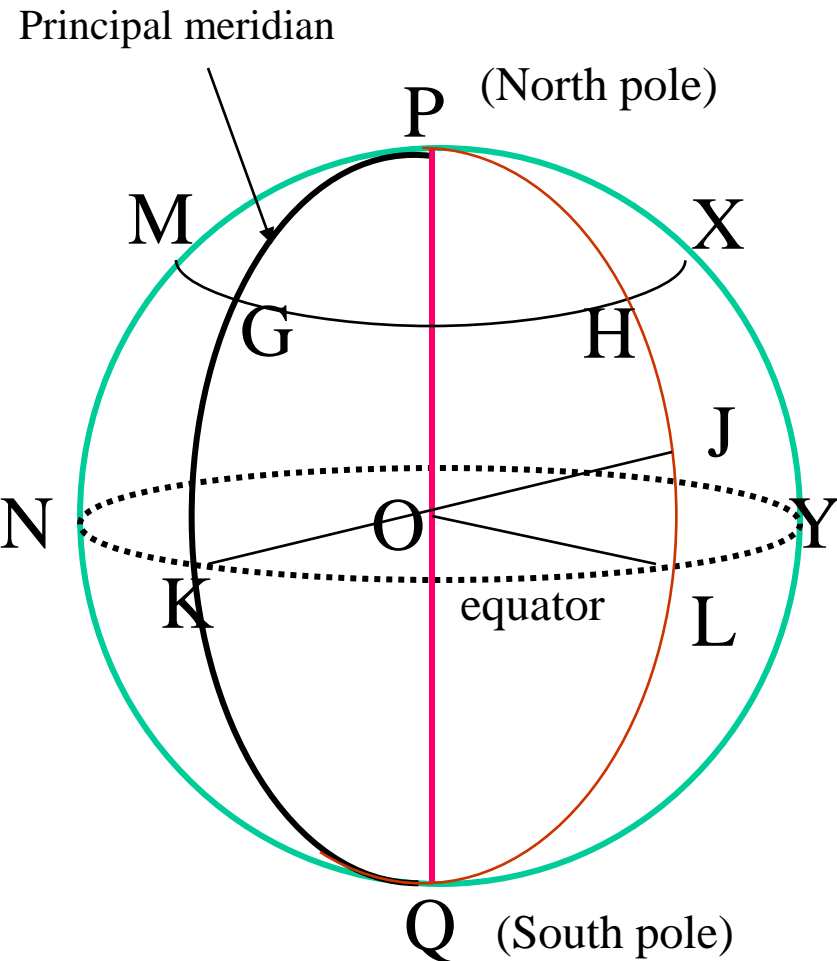
$$\underline{HX = LY \cos\theta}$$

where θ is the latitude of the Greenwich.

Nautical mile:

Defined as the distance between two points subtending an angle of one arc minute at the center of the earth. Thus

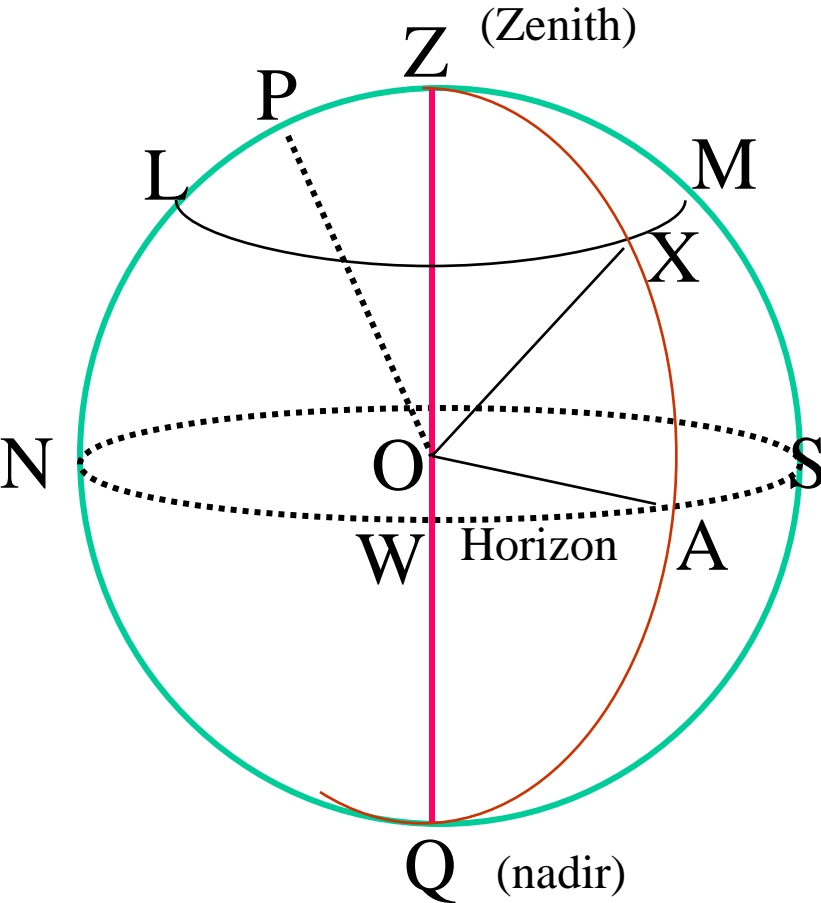
$$1^\circ = 60 \text{ nautical miles (one NM = 6080 feet)}$$



The celestial sphere: Altitude and Azimuth

Point Z on the CS vertically over head at point O (observer) is called **zenith**.

The plane through O at right angles to OZ is the plane of horizon, cutting the celestial sphere at NAS, called **celestial horizon or the horizon**

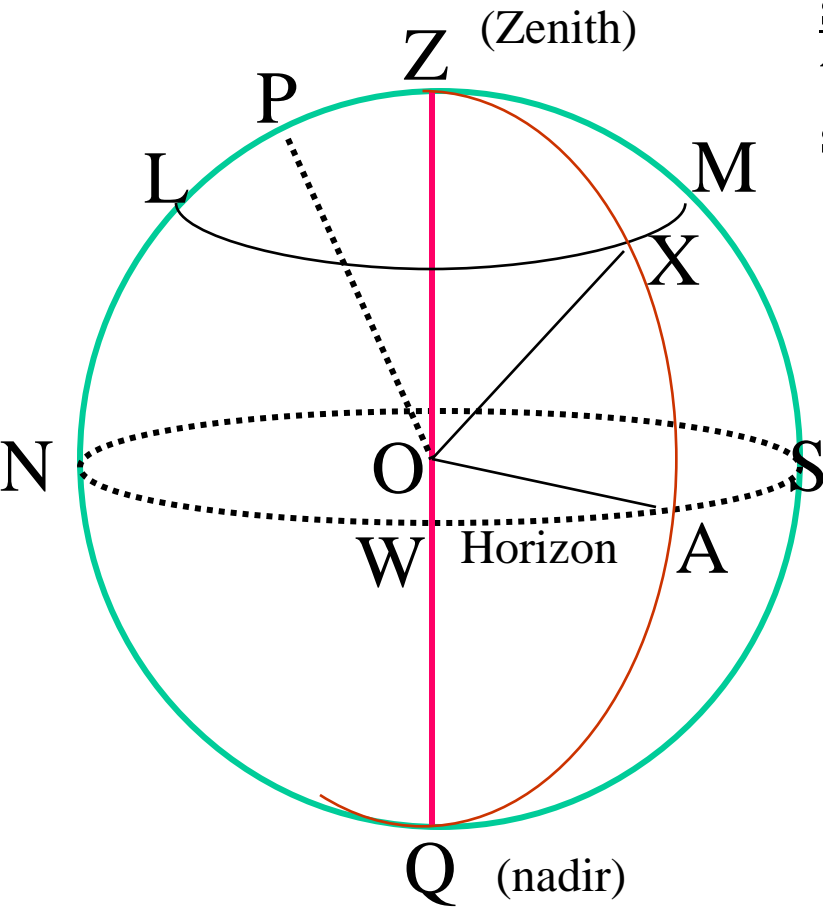


Any great circle passing through Z is called vertical circle. Here, ZXA is the vertical circle. Let X be the position of star at any given moment. In the plane ZXA, angle AOX or the great circle arc XA is called the altitude of X.

Since OZ is perpendicular to OA the zenith distance,

$$\underline{\underline{ZX(Z) = 90^\circ (ZA) - \text{altitude (XA)}}}$$

The celestial sphere: Altitude and Azimuth



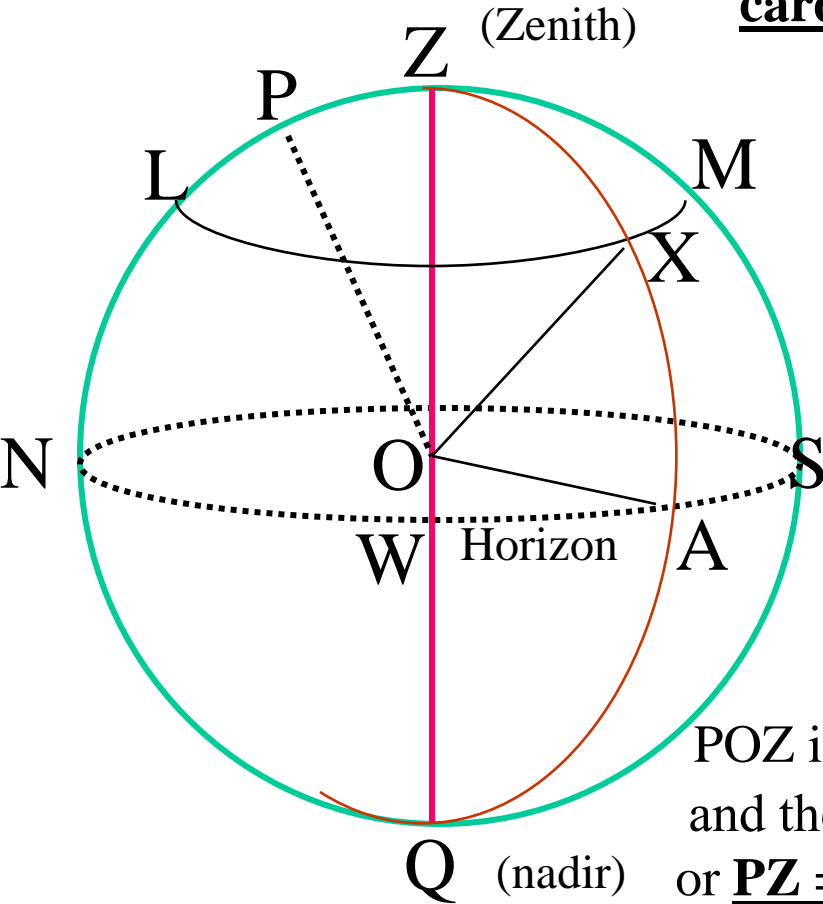
LXM is a parallel of altitude. All the stars which lie on this small circle have same altitude and also same zenith distance. Thus, if one knows 'z' or the altitude of a star, its parallel of altitude is specified.

OP is parallel to the spin axis of the earth. The point 'P' is called **celestial north pole** or the north pole. Stars around north pole move very little/none relative to O unlike stars over head.

A star which least changes its altitude or the direction relative to O is called **north pole star or Polaris.** Thus, position of Polaris is Invariable through out the night.

The celestial sphere: Altitude and Azimuth

Vertical circle through 'P' I.e., ZPN is defined as **principal vertical circle**. The point N is north point of horizon, similarly S, W, and E. Also, called **cardinal points**.



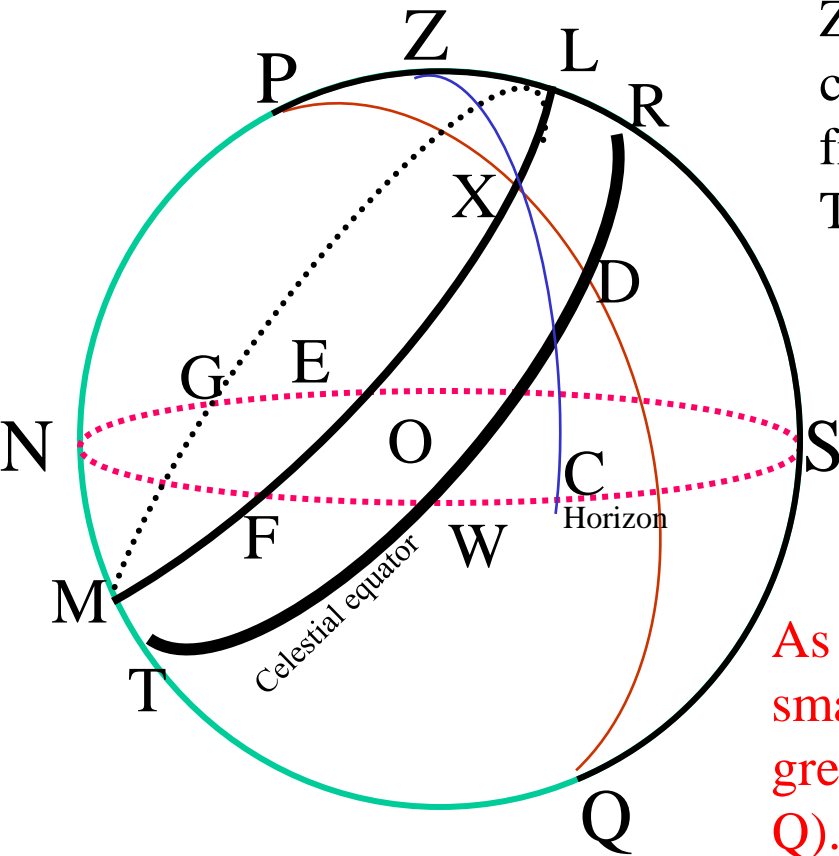
Azimuth:

X is the position of star on any vertical circle (ZXA). Angle PZX or the great circle arc NA is defined as **Azimuth** of X east or west depending on X's position. When the azimuth is 90° E or 90° W it is said to be on **the prime vertical** i.e., it is vertical circle passing either east point E or the west point W.

POZ is the angle between the radius of the earth (OZ) and the earth's axis OP is called colatitude of the observer or **$PZ = 90^\circ - \phi$** , where ϕ is latitude of the observer. Thus, **$PN = 90^\circ - PZ = \phi$** ; Thus **altitude of the pole star is equal to the observer's latitude.**

The celestial sphere: Declination and Hour angle

The great circle RWT perpendicular to OP is called **celestial equator** which is parallel to the earth's Equator.



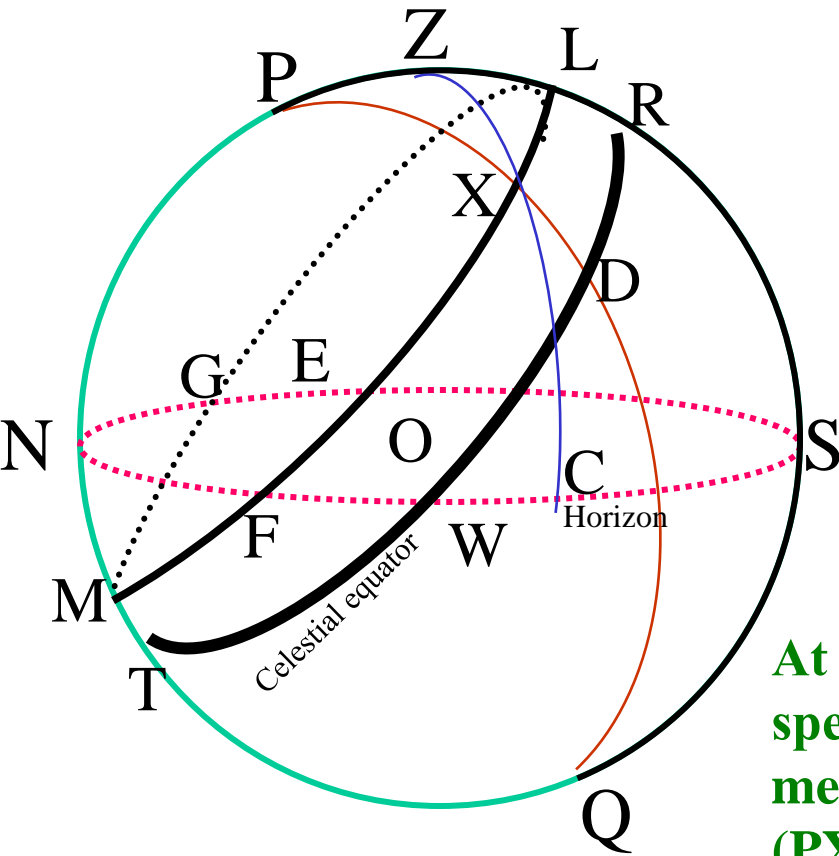
Z is the pole of the horizon and P is the pole of the celestial equator. W and E are cardinal points 90° from all points on the great circle through Z and P. Thus, $NW = SW = EN = ES = 90^\circ$.

Since stars are at great distances the angle between the straight line joining observer and any star, and the line OP (\parallel to earth's axis) remains unaltered.

As the earth rotates about OP any star X makes a small circle LXM \parallel to the CE. PXDQ be the semi-great circle through X and the poles of the CS (P and Q). Then the angle DOX or the arc DX is defined as the declination (δ) of the star. Hence, north polar distance of the star is $PX = 90^\circ - \delta$.

The celestial sphere: Declination and Hour angle

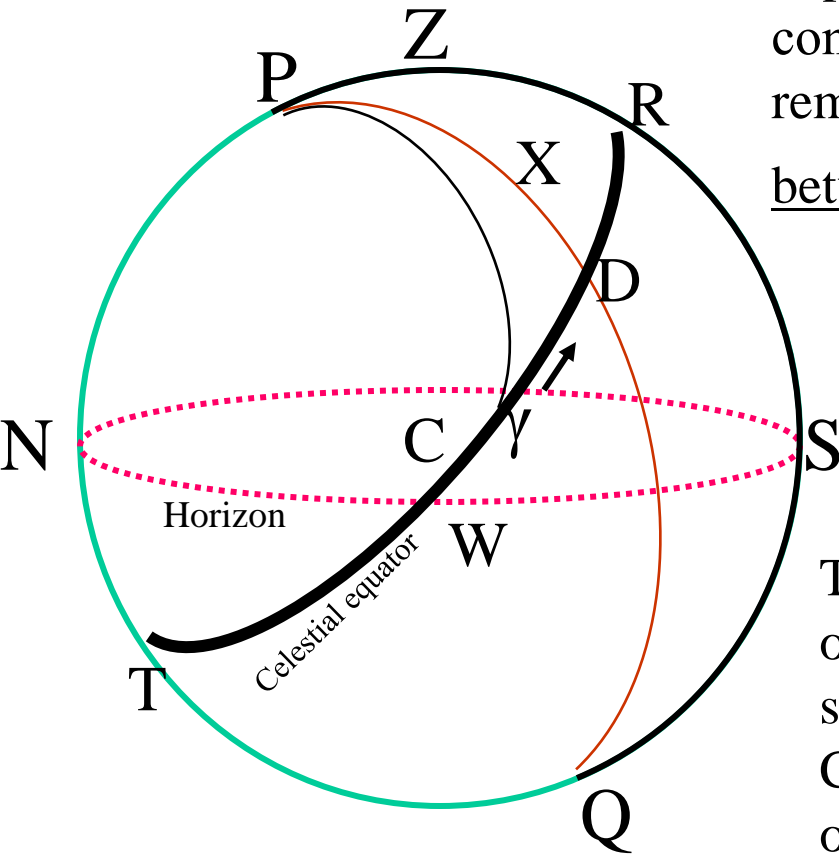
The small circle LXM is called P **parallel of declination**. All points on this have same δ .



PZRSQ is a semi-great circle called **observer's meridian**. Star (L) on the observer's meridian is said to be at transit. Star at L is at its greatest altitude and at least zenith distance (ZL). Its altitude keep changing and reaches horizon at F and minimum at M. Again at G star rises and reaches high at L. Thus, star from L describes 360° along a small circle LXM.

At any moment star's position on LXM is specified by the angle at P between the observer's meridian and the meridian through the star (PXQ). Thus, the angle RPX or the arc RD on the equator is called hour angle (H). H is measured from O – 24 hours (0- 360°) westwards.

The celestial sphere: Right ascension and Declination



Let γ be an equatorial star and X any other star on the meridian through X cutting CE at D. As X passes across the sky, we know its δ or DX is constant and also X's relative position with γ remains same. Thus, γD is constant i.e the angle between the meridians of γ and D remains constant.

Therefore, with respect to γ as a reference point on the CE the position of X is completely specified. The angle γPX or the arc γD on the CE is defined as right ascension (α). The point of reference γ is called the vernal equinox or the first point of Aries.

The celestial sphere: Right ascension and Declination

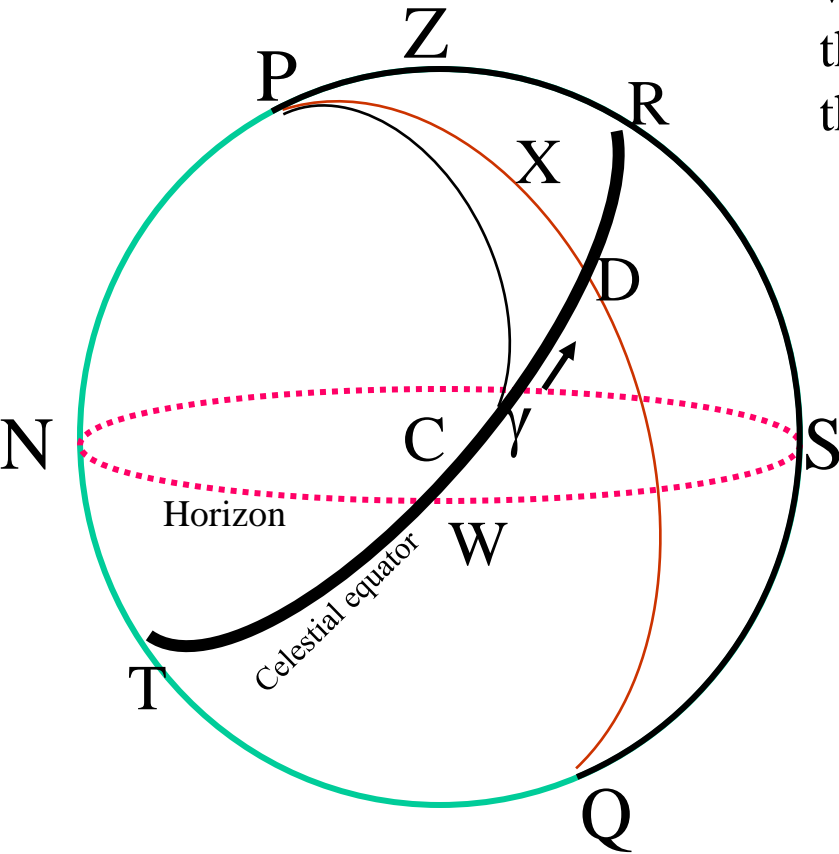
The α is measured eastwards from Oh – 24h. And,

$$\mathbf{R\gamma = RD + D\gamma};$$

where RD (RPX) is the hour angle of X and R γ is the hour angle of γ . The hour angle of γ is called the sidereal time (S.T).

$$\mathbf{S.T = H.A of X + R.A of X}$$

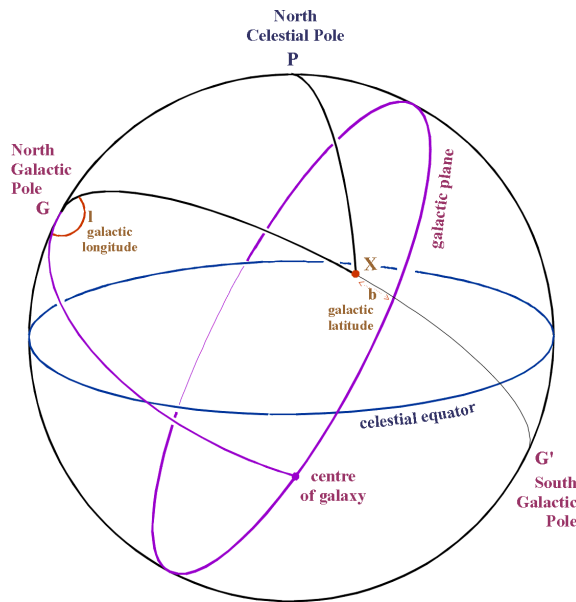
$$\mathbf{S.T = H + \alpha}$$



Thus, when γ is on the Observer's Meridian (PZRSQ), the HA of γ is 0h and S.T is 0h. When γ is next on the OM, 24h elapsed. This is the time required to complete one rotation of the earth about its axis. This time is called a sidereal day.

Galactic Latitude and Longitude

Here, fundamental great circle is the galactic equator with the intersection of the galactic plane with the celestial sphere. G and G' are the north and south galactic poles (after IAU).



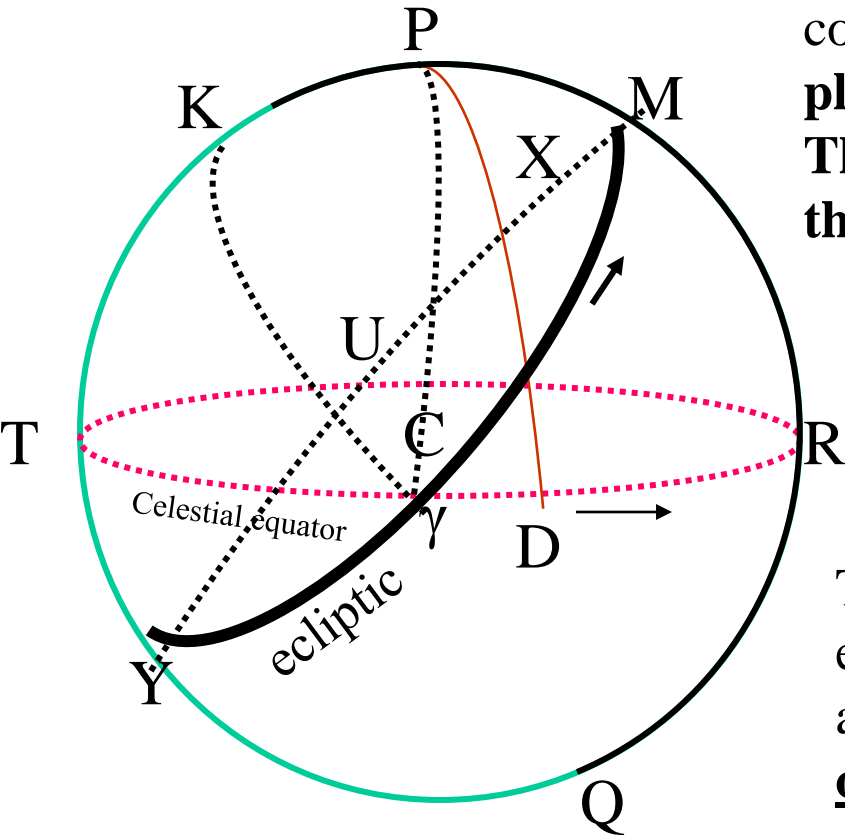
The galactic latitude (b) of a star X on this great circle GXG' is the angular distance from the galactic Equator to X. This is measured from -90° at south galactic pole to $+90^\circ$ at north galactic pole.

Zero point for longitude is the center of the galaxy. **The longitude** (l) of star x is the angular distance from the center to the great circle GXG'. This is measured from 0-360°.

Further study: conversion of Galactic coordinates (l, b) to equatorial coordinates (α, δ). See Binney and Merrifield (Galactic Astronomy)

The earth's orbit: The ecliptic motion

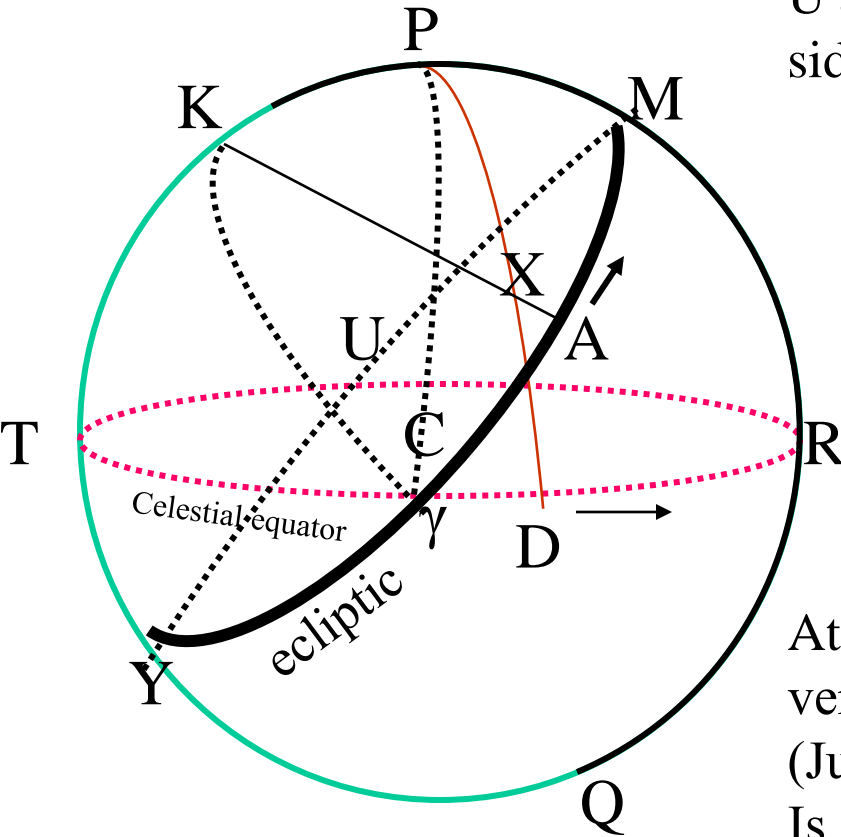
Imagine the earth is at the center of celestial sphere. The Sun appears to make an elliptical Orbit around the earth. The sun makes one complete circuit of the heavens in one year. **The plane of the orbit is called the plane of ecliptic. The great circle in which this plane intersects the celestial sphere is called the ecliptic.**



T γ R is celestial equator and Y γ M is the ecliptic. The ecliptic is inclined at an angle of about 23.5° to the CE. Angle M γ R is called the obliquity of the ecliptic.

The earth's orbit: The ecliptic motion

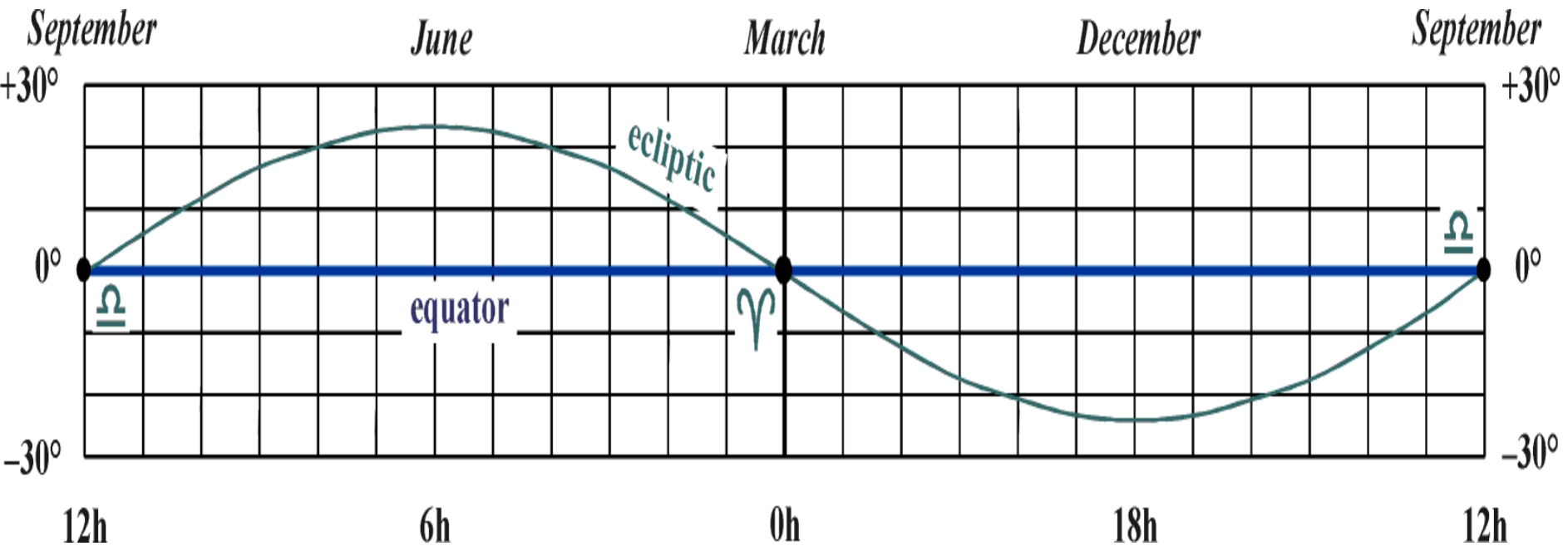
Ecliptic touches the celestial equator twice a year at γ and U. Between γ and M and M and U the sun is on the north pole side of the equator and between U and Y and Y and γ the sun is on the south pole side of the equator.



The position γ , at which the sun's declination changes from south to north, is the vernal equinox. RA of the star (X) is measured with reference to γ along the CE eastwards. γD is X's RA and DX is its δ .

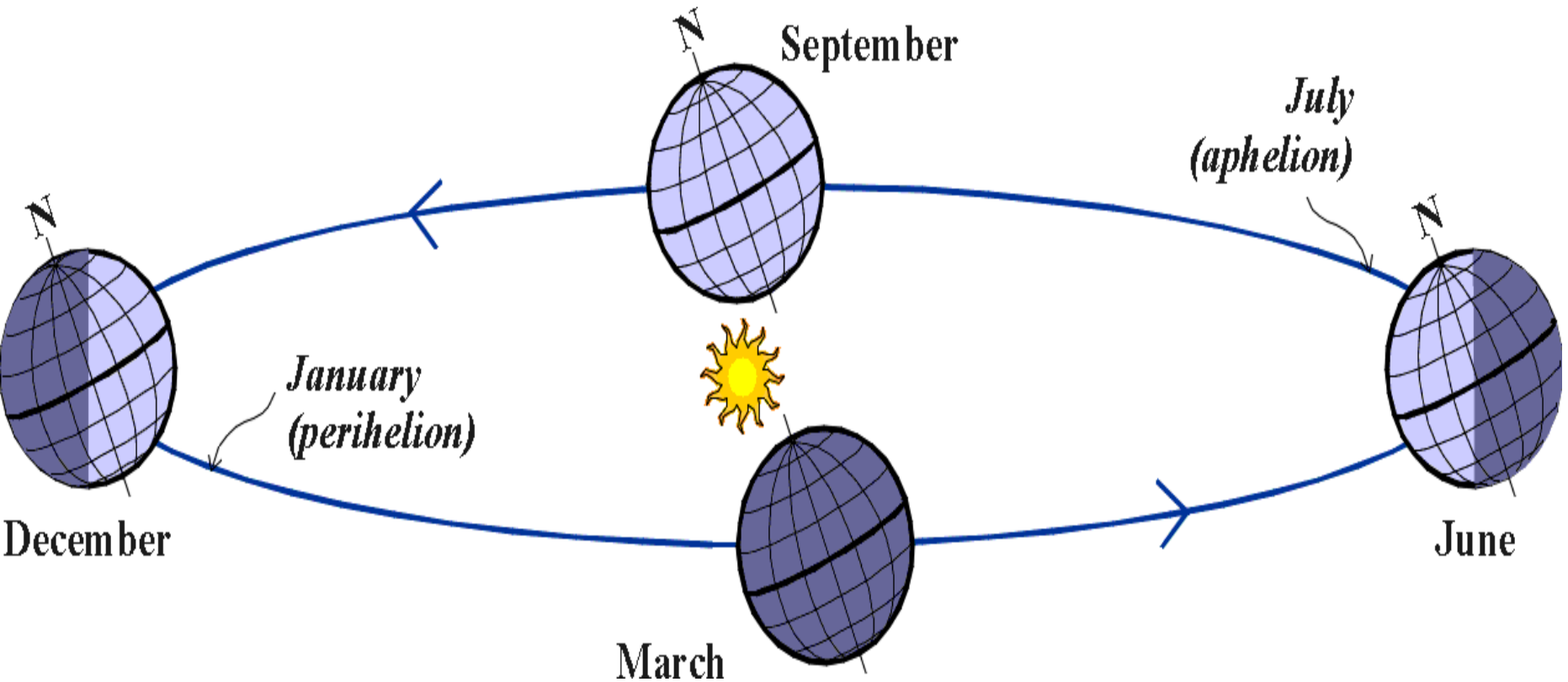
At γ sun's RA and δ are zero (March 21) called vernal equinox. At M RA is 6h and δ is 23.5° N (June 21: summer solstice). At U RA the sun's RA is 12h and δ is zero (Sept. 21: autumnal equinox). At Y RA is 18h and δ is 23.5° S (December 21: winter solstice).

The earth's orbit: The ecliptic motion



At γ sun's RA and δ are zero (March 21) called vernal equinox (march 21).
Three months later sun is 6h from γ and δ is 23.5° N (June 21: summer solstice).
Six months later sun is 12h (its RA) from γ and its δ is zero (Sept. 21: autumnal equinox).
Sun's RA is 18 h and its δ is 23.5° S on December 21 called winter solstice.

Seasons



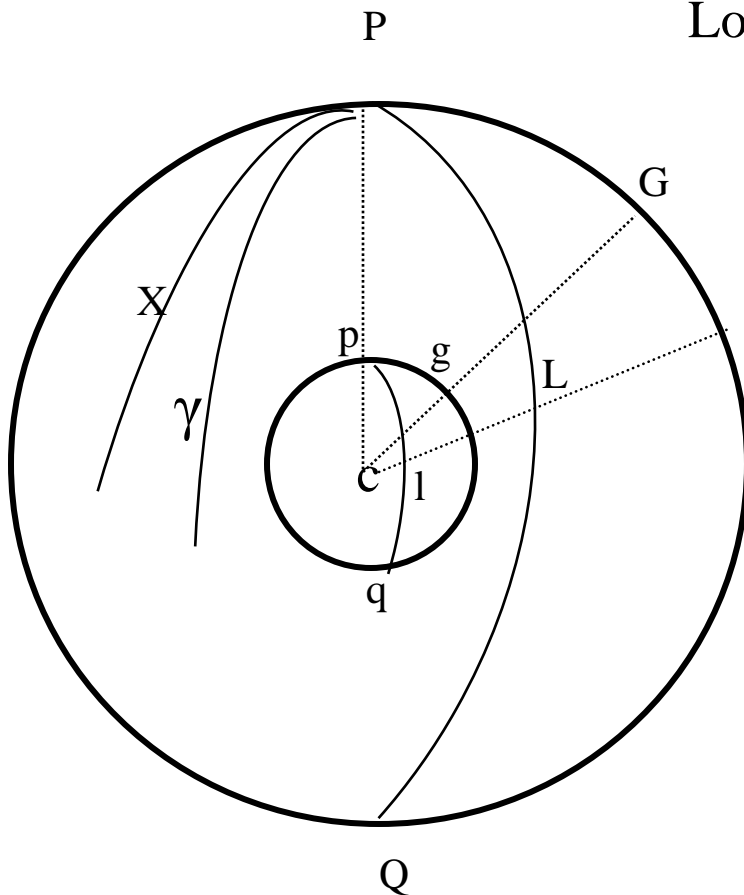
The most northerly point of the ecliptic is called, in the northern hemisphere, the summer solstice (RA=6h). At this point the Earth is topped towards the Sun giving longer hours of daylight and warmer weather.

The most southerly point is the winter solstice (RA: 18h) giving longer night hours and colder weather for northern parts of the globe.

Sidereal Time

C is the center for both the Earth and the CS. g is the position of Greenwich and l is any other Place on earth.

Longitude on earth is the angle between plq and pgq.



Extend Cg and Cl to produce CG and CL so that meet CS. Now G and L zenith positions of an observer at g and l on the earth's surface, respectively.

For a star X the angle GPX is the hour angle for an observer at g and angle LPX is the hour angle of star X for an observer at l.

Sidereal Time

$$\text{Angle GPX} = \text{angle LPX} + \text{angle GPL} (= \text{angle gpl})$$

$$\text{H.A of X at g} = \text{H.A of X at l} + \text{longitude (w) of l}$$

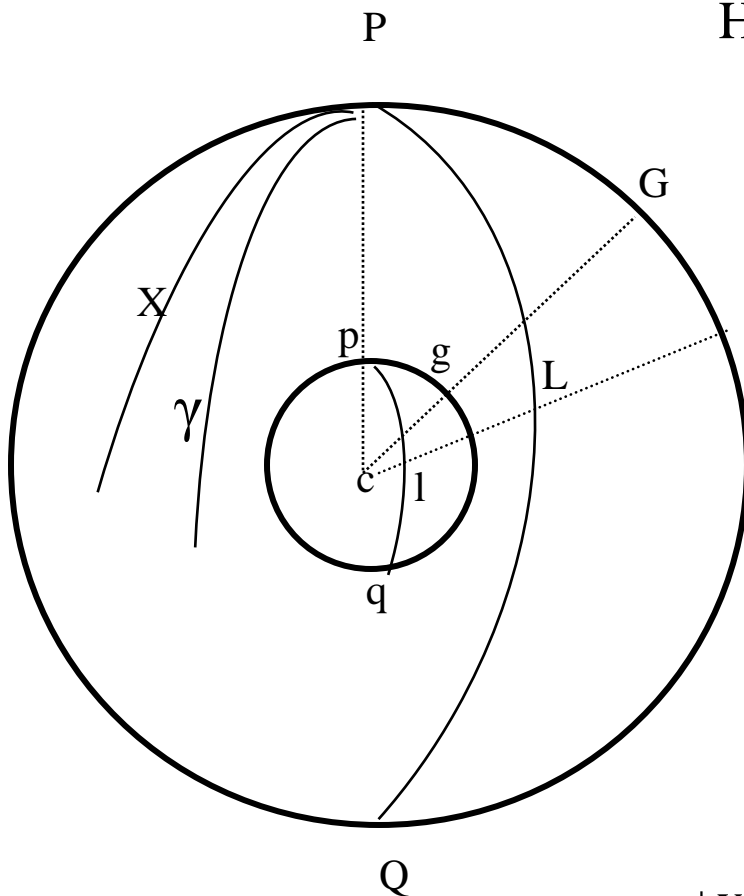
This also holds good for the equinox and the sidereal time is H.A of γ

$$\text{ST at g} = \text{ST at l} \pm \text{long. of l}$$

or

$$\text{ST. at l (LST)} = \text{ST at g (GST)} \pm \text{long of l}$$

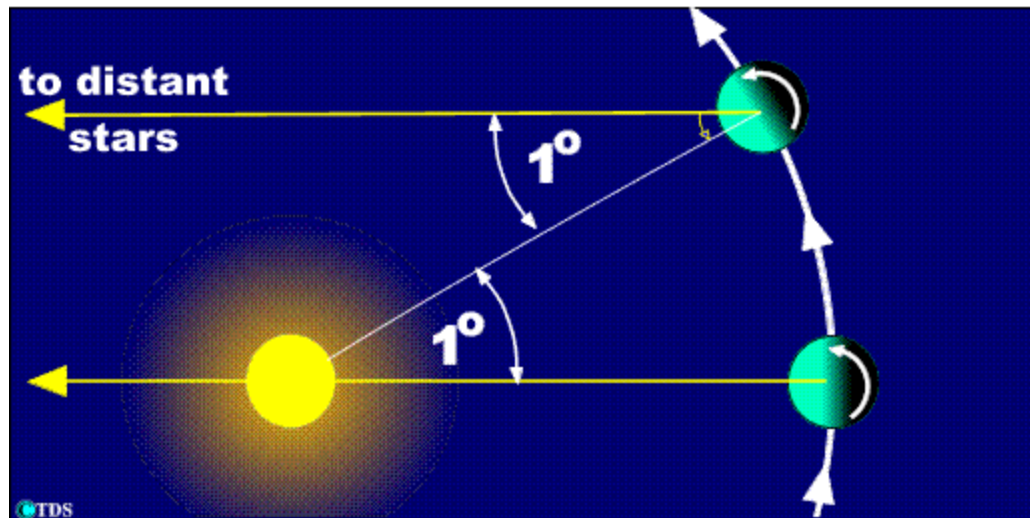
+ve for east longitudes and -ve for west longitudes



Sidereal Day

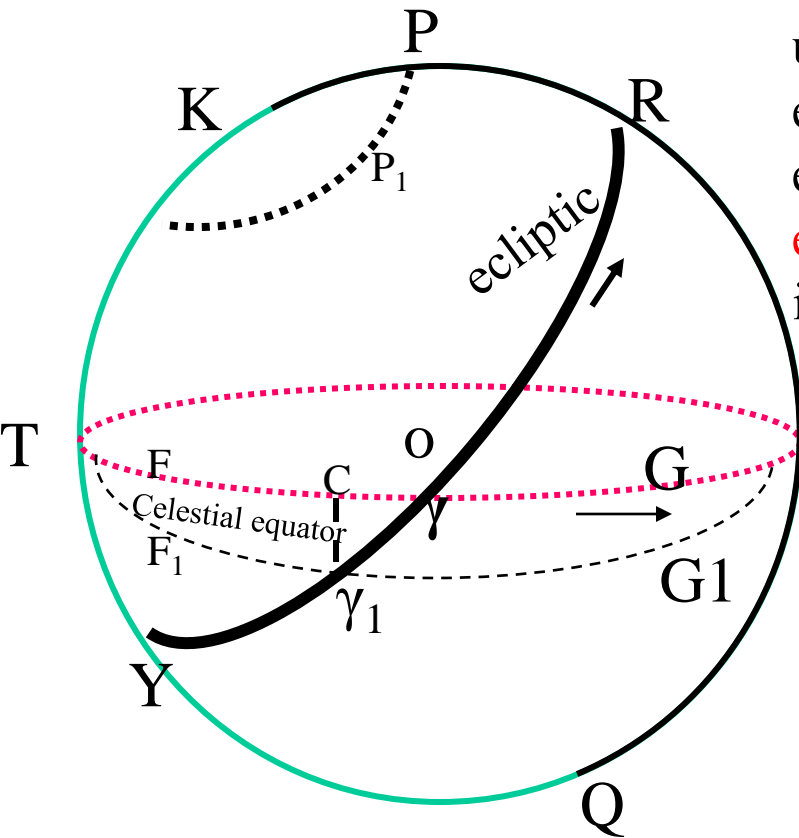
The time it takes for the Earth to spin 360 degrees with respect to the star. This has been found out to be 23 hours, 56 minutes.

Solar Day: It is also found the Earth takes 24 hours to spin 360 degrees With respect to the Sun.



Precession

F γ G is the CE with north pole at P. One year later F1 γ G1 is the CE corresponding to the north pole position at P1. Similarly, γ and γ 1 are vernal equinox Positions for year 2000 and 2001, respectively.



Due to the precession the north celestial pole moves uniformly along the small circle arc PP1 and the equinox moves uniformly backwards along the ecliptic from γ to γ 1. **The movement of γ along the ecliptic is called the precession of equinox.** This is found to be at the rate of $50''.3$ per year.

This has an effect on star's RA and Dec. Let C γ 1 be a great circle arc right angles to CE F γ G.

γ C = γ γ 1 $\cos \epsilon$, where ϵ is the obliquity of the ecliptic. Thus,

$$\gamma$$
C = $50''.3 \cos \epsilon$

One may find rate of change in RA $0.008s$ per sidereal day

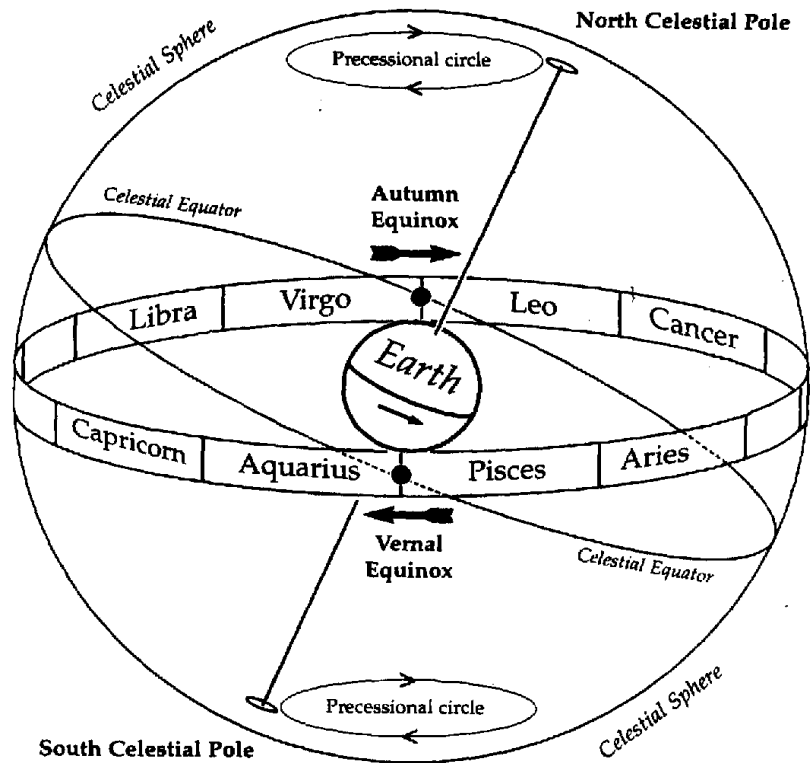
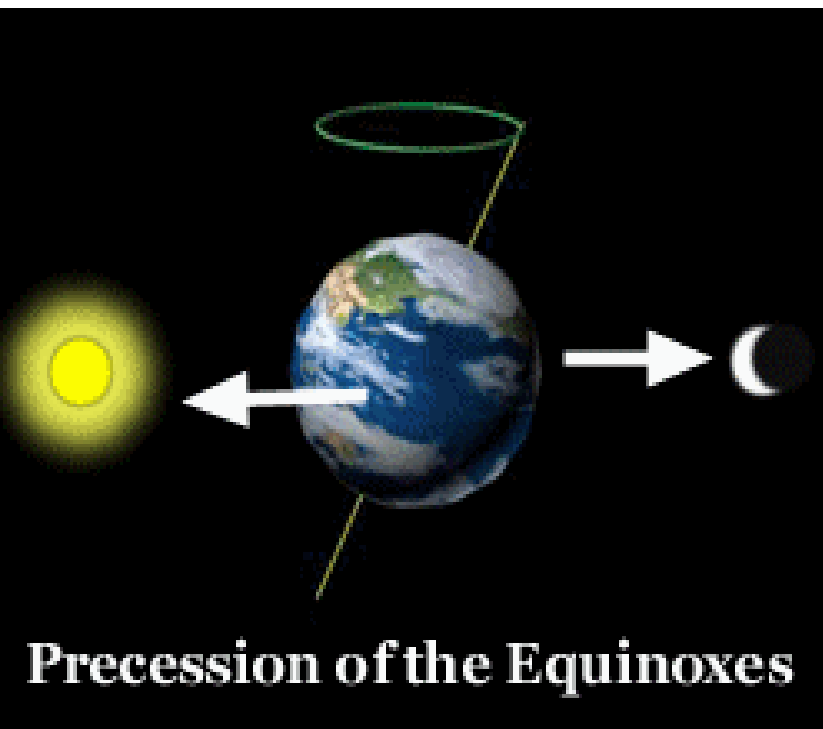
Precession

For a star of (α, δ) at one epoch and (α_1, δ_1) at another epoch the change in RA and Dec are related as

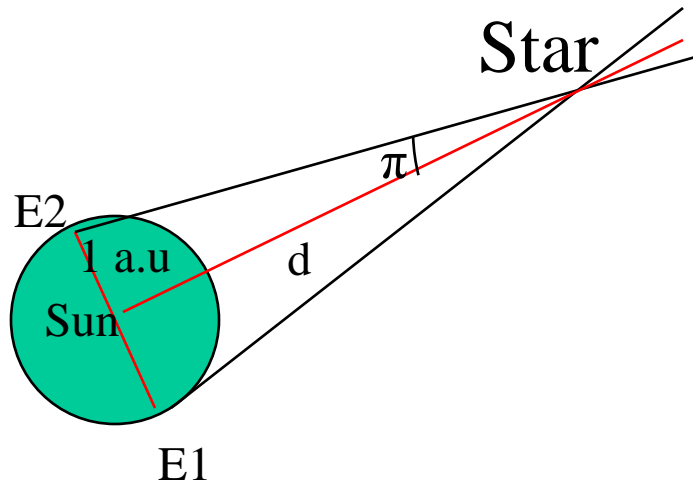
$$\Delta \delta = \psi \sin \varepsilon \cos \alpha$$

$$\Delta \alpha = \psi (\cos \varepsilon + \sin \varepsilon \sin \alpha \tan \delta)$$

Where, ψ is yearly rate of change in RA, and ε obliquity ($23^\circ 27'$)



Distances: Parallax



At a point E1 on the Earth, the direction to a star S is along the line E1S and at point E2 (six months later) its direction is along E2s.

During the course of the year, the apparent position of a star traces out an elliptical path called the **parallax ellipse**. Lines E2s and E1s contain an angle π .

Let r be radius of the earth orbit
and d is the distance to the star
Then,

$$\tan(\pi) = r/d \text{ or } d = r/\tan(\pi) \text{ radians}$$

Or $d = r/ \pi$ radians (as the angle is very small).

Converting radians into arc seconds (2π radians = 360°)

$$\pi'' = 206265 \pi \text{ radians}$$

$d = 206265 r/ \pi''$ (define one AU so that $r=1$ AU),
thus

$$d = 206265/ \pi'' \text{ AU}$$

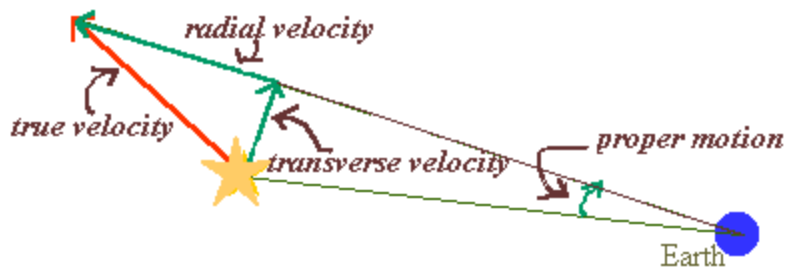
Parsec (pc) is the distance at which a star would have a parallax of $1''$.

$$1\text{pc} = 206265 \text{ AU} = 3.086 \times 10^{13} \text{ km} = 3.26 \text{ light years.}$$

$$d = 1/ \pi'' \text{ pc}$$

The distance to the Hyades cluster

Figure 1: *The components of the velocity of a star.*



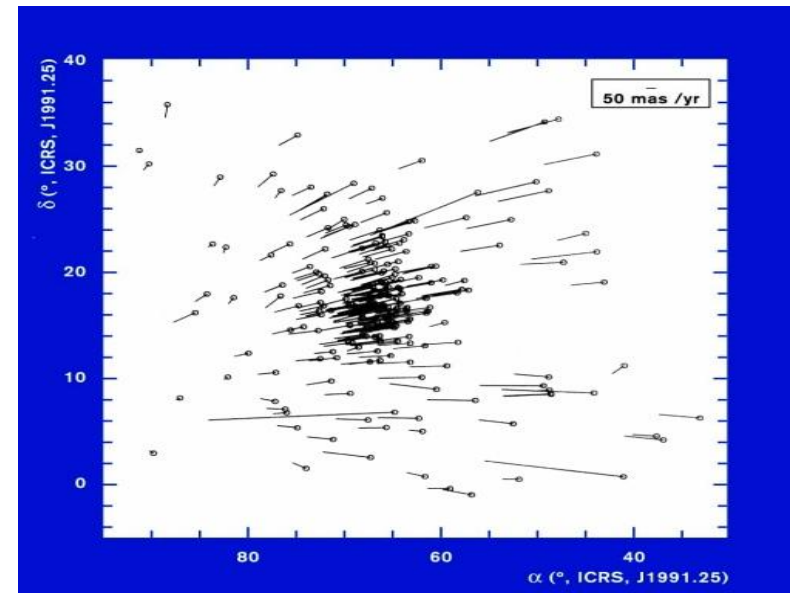
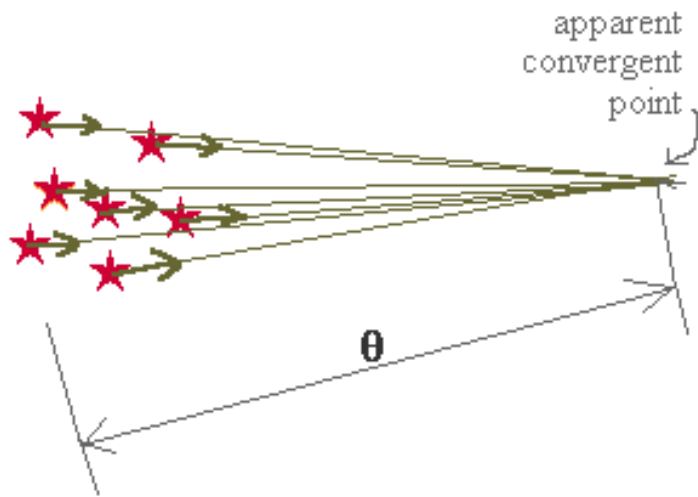
For stars that are too far away to measure a reliable parallax we can apply a different geometrical method if those stars belong to a close cluster. We measure the distance from an Actual radial velocity and a proper motion

$$\text{Radial velocity } V_r = C \delta\lambda/\lambda$$

The distance to the Hyades cluster

If a group of stars is moving together, we can sketch the motion of each star in space as shown. The point of convergence is determined on a chart of the sky by simply extending the lines of proper motion of each star and finding their point of intersection.

Figure 2: *Motion of cluster stars in space.*



The distance to the Hyades cluster

θ is the angle between the space velocity (v) and the radial velocity v_r . Thus, the transverse velocity is given as

$$v_t = v_r \tan \theta$$

Relationship between distance traveled (x), velocity (v_t) and time (Δt) is

$$x = v_t \Delta t$$

Since D is very large $\tan \Delta \theta = \Delta \theta$, and the relation between D and x is written as

$$\frac{D \text{ (km)}}{206265 \text{ arcsec/radian}} = \frac{v_t \text{ (km/sec)} \Delta t \text{ (sec)}}{\Delta \theta \text{ arcsec}}$$

$$D \text{ (pc)} = \frac{v_t \text{ (km/s)}}{4.74 \mu \text{ (''/yr)}}$$

Where, $\Delta \theta / \Delta t = \mu$, expressed in arcsec/yr.

$$\text{Pc} = 3.09 \times 10^{13} \text{ km, radian} = 206265 \text{ arcsec, 1year} = 3.16 \times 10^7$$

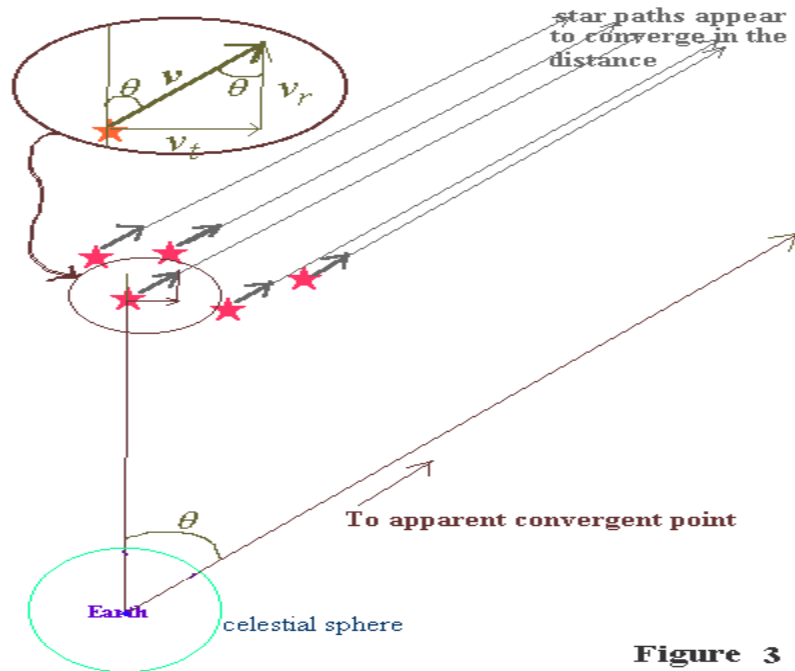
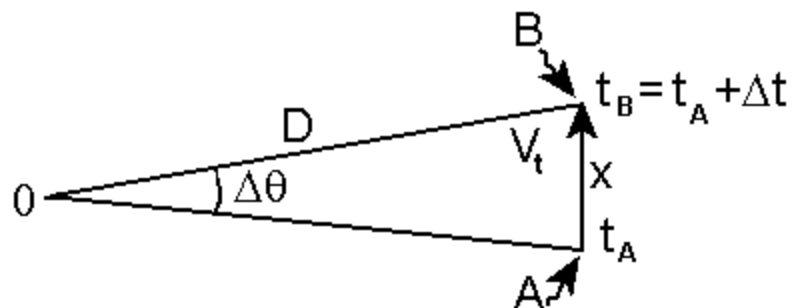


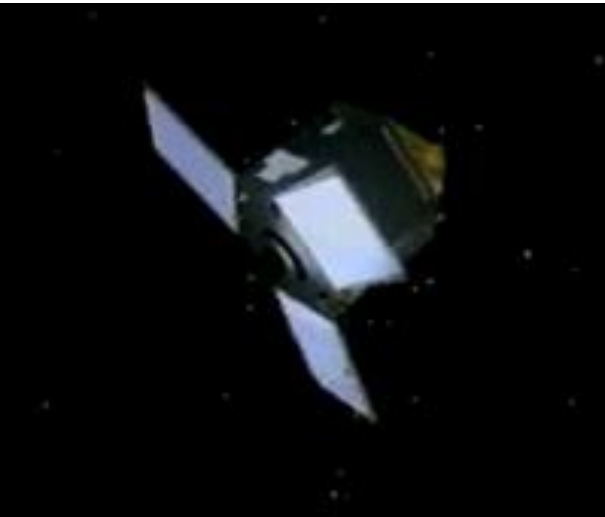
Figure 3



Distances: Parallax

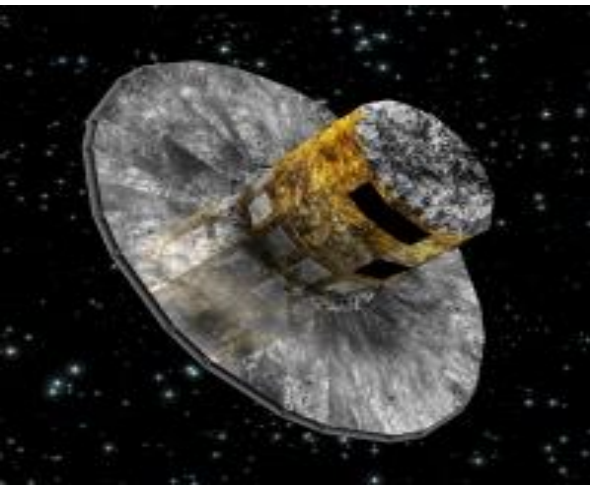
Star's parallax is the reciprocal of its distance.

Farther the star smaller is the parallax.



F.W. Bessel in 1838 was the first to measure parallax for star 61 Cygni which is 0.29''.

Hipparcos (High precision parallax collecting satellite) satellite in 1989 (launched by ESA) measured accurate parallaxes for about 120,000 stars upto distances of 1kpc with errors of 10% for stars below 100pc. The catalogue was released in 1997.

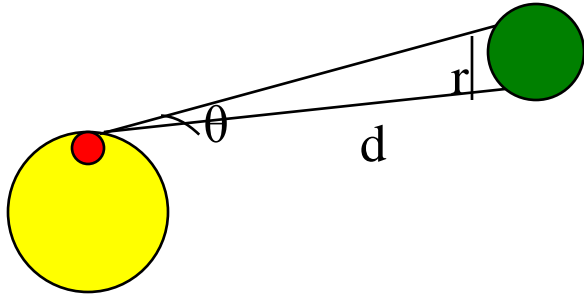


Gaia is European space mission to measure astrometry for one billion stars in our Galaxy. Gaia will measure stars down to $m_v = 20$. It measures 24microns at $m_v=15$ implying measuring the diameter of a human Hair from 1000km. Gaia will be launched in Dec 2011 and data is expected from 2016.

Distance to the Sun: What is 1 AU in Km?

Venus transit:

$$\tan(\theta) = r/d \text{ km.}$$



$$\theta = 8.794''$$

The sun is too bright to see background stars. Base line of the Earth's diameter is small compared to the distance to the sun.

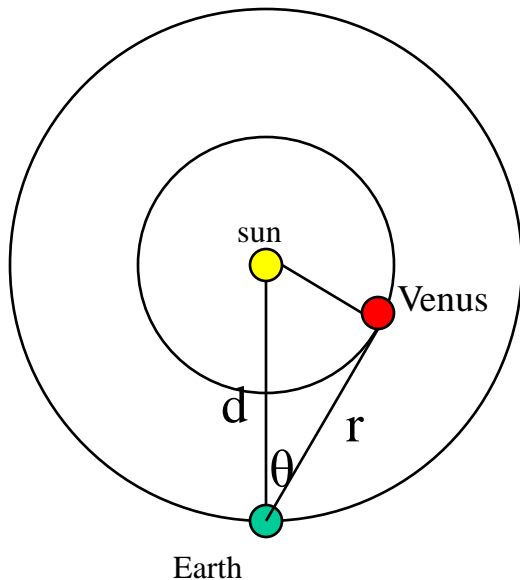
Method a: timing the passage of either Venus or mercury across the sun's disk from two well separated places on the Earth. Timing should be Synchronized. **Difference in timings between the two observations of start or end of the transit is the angle at the Sun between the directions of two observers.** Separation between the two observing locations on the earth in km is known. But transits are very rare. Recent one was 2004 (earlier one was in 1882).

Distance to the Sun: What is 1 AU in Km?

Method b: Venus orbits inside the Earth's orbit. We observe Venus to make angular separation from the sun (called elongation). This varies from 0° to some maximum. At maximum the directions of Venus – sun and earth – Venus form right angles.

Maximum elongation:

$$\cos(\theta) = r/d \text{ or } r = \cos(\theta) \times 1 \text{ au}$$
$$\text{or } r = \cos(46.3^\circ) \times 1 \text{ AU} = 0.6905 \text{ AU}$$



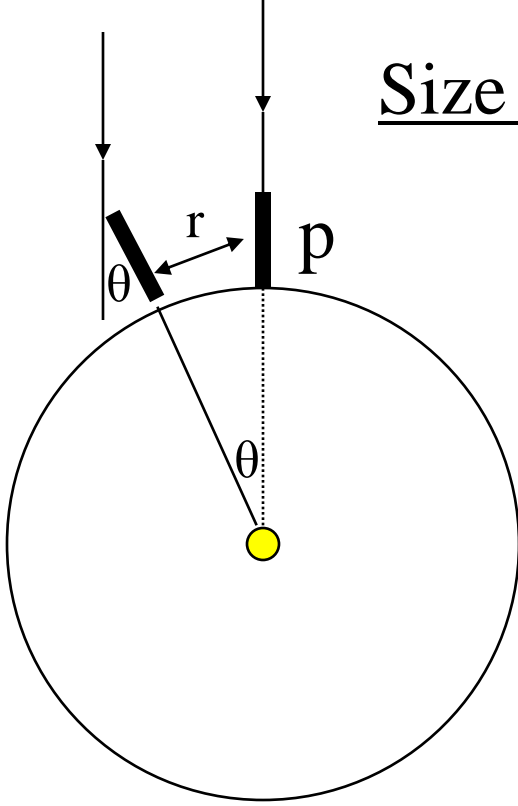
Next is to measure Venus's distance using parallax method with the aid of background stars. Or use a radar beam and measure the time for the round trip.

Thus, $r = \text{speed of light } (c) \times t/2 \text{ (km)}$
where t is the time elapsed.

Therefore, $c \times t/2 = 0.6905 \text{ AU}$

$$\underline{\underline{1 \text{ AU} = 1.4959 \times 10^8 \text{ km}}}$$

Size of the Earth

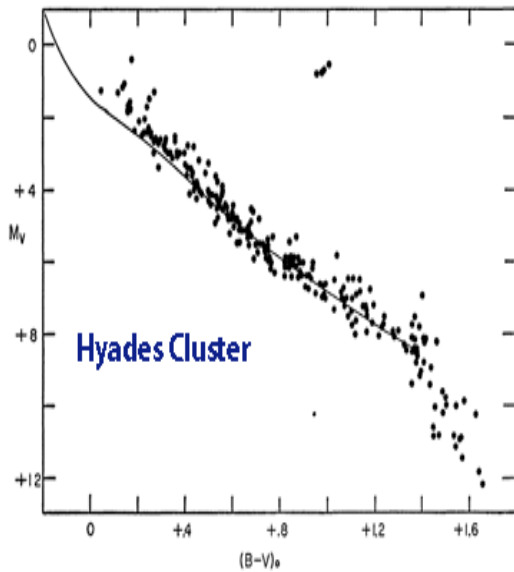


In 200 B.C Eratosthenes attempted to measure Earth's size and he could get circumference, $c=40,000$ Km. Today's values are 40,075 km along the equator and 40,008 km through the poles.

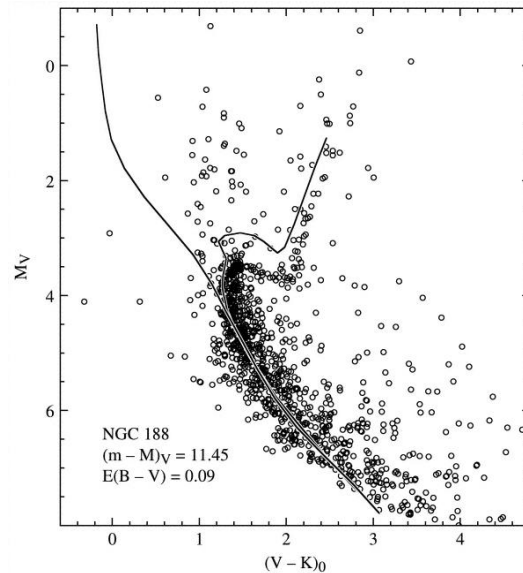
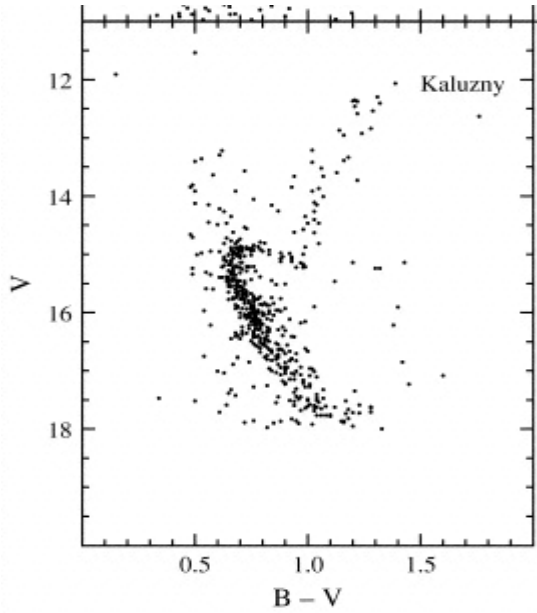
$$\Theta/360^\circ = r/c$$

It was just overhead in Syene in Egypt. At the same time in Alexandria he measured the angle 7.5° made by the shadow. The cities are 5000 stadia (800 km) apart.

Stellar Distances: main sequence method



All stars in a cluster lie at the same distance allowing us to determine the distance by making an H-R diagram by plotting apparent magnitude and the colour (B-V or V-K) of stars. We can calculate the distance to the cluster by comparing with the standard H-R diagram of true luminosities and the colour. *The method of obtaining the cluster distance is called main sequence fitting.*

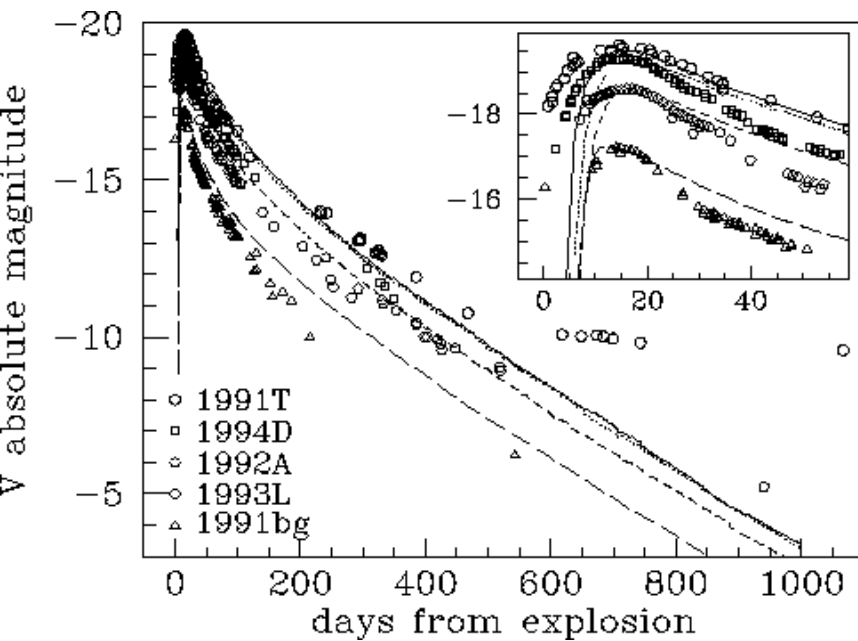


$$(m-M)_V = 5 \log(d/10)$$

Or

$$d = 10^{((m_V - M_V + 5)/5)}$$

SN1a: distance indicators



They are luminous

Same peak amplitude with little dispersion

Physics of SN1a understood well

Local tests for calibration.

Problem: prediction of SN explosion

1-2 per galaxy in a millenium

Cepheid variables: distance indicators

Henrietta Leavitt was first to notice that the cepheids in Large Magellanic clouds have relationship between their apparent brightness (mv) and the period. **Brighter Cepheids have longer pulsation periods**. Since all the stars in the LMC are at the same distance from us, the period must be related to the luminosity.

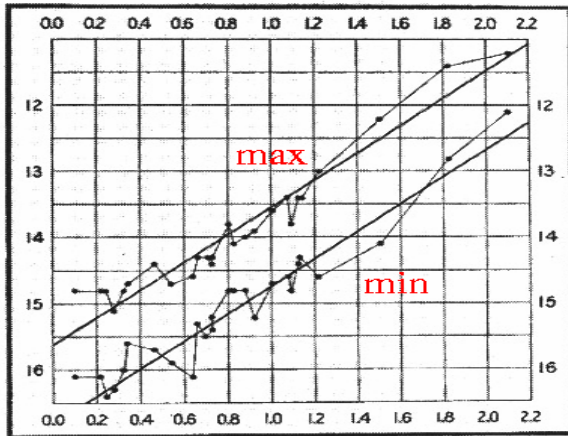
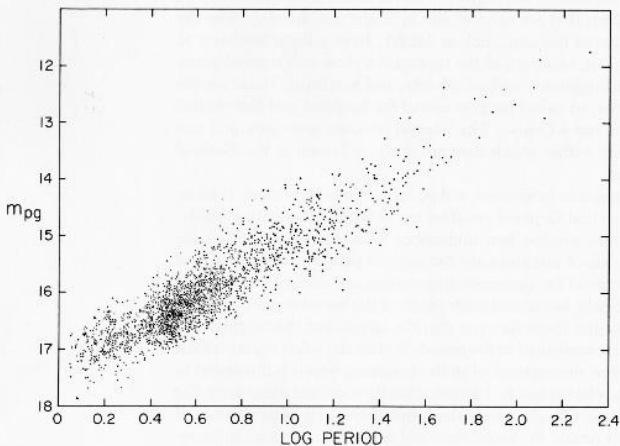


FIG. 2.



Red shifts (Z): cosmic distance scales

$$\text{Red shift (z)} = \delta\lambda / \lambda_0$$

Where, λ_0 is the rest wavelength

Hubble law states that farther the galaxy larger is the value of z, which is

$$D = cz / H_0$$

Where, H_0 is the Hubble constant and c is the Velocity of the light.

$$H_0 = 100 h \text{ km s}^{-1} \text{ Mp c}^{-1}$$

where, h is 0.5 to 1.0. One also make an estimate of age of the universe using $t = D/v$ as $v = cz$ (for $v \ll c$)

Proper motion

Stellar Radii

Stellar Masses

Stellar magnitude system

Stellar magnitude system predates the telescope and is based on human eye.

Hipparchus, the Greek Astronomer, in 200 BC, first divided the naked eye stars into six brightness classes. He produced a catalogue of 1000 stars ranking them by “magnitudes” one through six. The first magnitude star is the brightest and the sixth one is the faintest to the eye.

Pogson (1857) (the director of the Madras Observatory) placed the magnitude on precise scale. It was observed that a first magnitude star is about 100 times brighter than the 6th magnitude i.e., 5 magnitudes lower.

Thus, the light flux ratio for a one-magnitude difference was found to be $100^{1/5}$ or 2.512.

One may write this as 1st magnitude star is 2.512^{2-1} brighter in intensity than the 2nd magnitude star, and 2.512^{3-1} than the 3rd magnitude star and so on.

Stellar magnitudes

$$f_1/f_2 = 2.5^{(m_2-m_1)} = 10^{0.4(m_2-m_1)}$$

$$\text{or } \log (f_1/f_2) = 0.4(m_2-m_1)$$

$$m_1-m_2 = -2.5 \log (f_1/f_2)$$

f_1 and f_2 are fluxes received on the earth which are functions of true brightness of star, its distance, extinction caused by intervening medium, wavelength and the detector

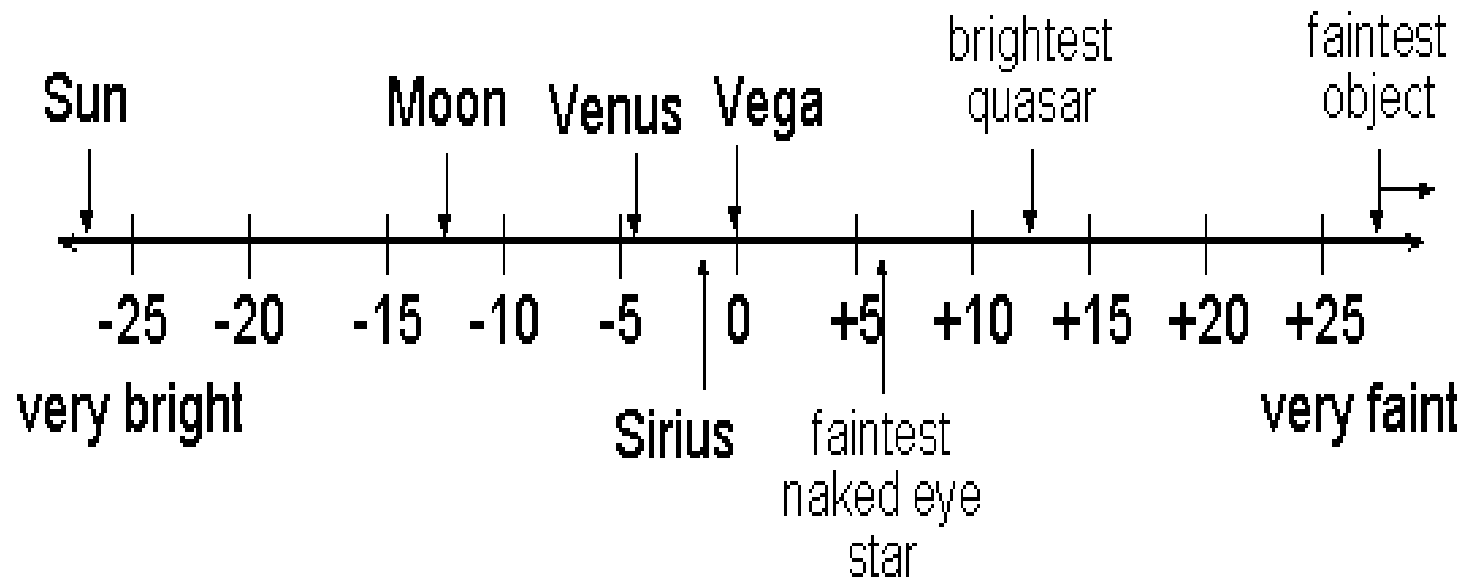
Note: $m_1-m_2 = 0.921 \log_e (f_2/f_1)$. The scale is defined by ancient Greeks was essentially based on natural logarithms.

Stellar magnitudes

$$f \propto 1/d^2$$

$$m_1 - m_2 = -2.5 \log (d_2/d_1)^2$$

This is useful to estimate distances to an unknown cluster or a star if the difference in magnitudes is given.



Apparent brightnesses of some objects in the magnitude system.

Absolute magnitudes

It is measure of the intrinsic brightness of a star and is
Defined as the apparent magnitude that a star has when
Placed at a distance of 10 parsec (pc) from the Earth.

Flux received on the Earth at distance 'd' and '10pc'
Is written as

$$f = (D/d)^2 F;$$

$$f(d) / F(10) = [10 \text{ pc}/d]^2; \quad \text{for } D = 10 \text{ pc};$$

$$m - M = -2.5 \log [f(d) / F(10)]$$

$$= -2.5 \log [10 \text{ pc}/d]^2$$

$$m - M = 5 \log d - 5$$

where, M is the absolute magnitude and the
quantity $(m - M)$ is called the distance modulus.

Absolute magnitudes

$$m-M = 5\log d - 5$$

If we know m and M we can estimate distance to the star

Absolute magnitudes are generally expressed as M_v . Note this is not total output energy of the star.

Distance modulus of the Sun is $m_v - M_v = -31.57$ and its $M_v = +4.83$.

Star light gets dimmer due to intervening ISM. Photons get scattered and/or absorbed by atoms and molecules in the space. Net effect of the dimming is expressed in magnitudes. Above expression is rewritten as,

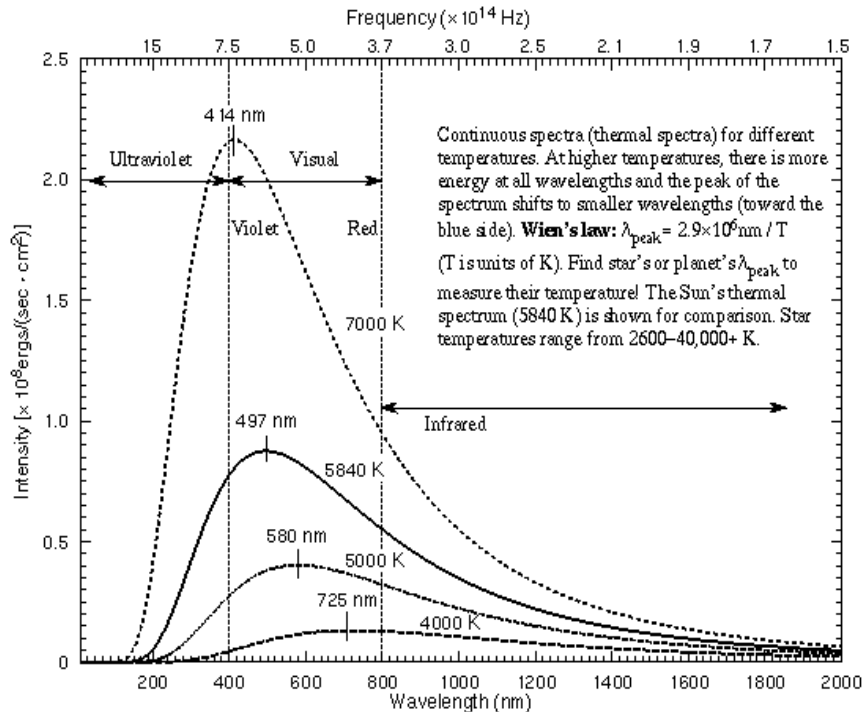
$$m-M = 5\log d - 5 + A$$

Where A is called **interstellar extinction** which is strong strong function of wavelength.

Bolometric magnitudes

Total measured radiation at all wavelengths from a star is defined as bolometric magnitude which is defined as m_{bol} and M_{bol} for apparent and absolute bolometric magnitudes, respectively.

If a star is a strong infrared or ultraviolet emitter, its bolometric magnitude will differ greatly from its visual magnitude. The *bolometric correction* (*BC*) is the visual magnitude of an object minus its bolometric magnitude.



$$M_{bol} = M_V + BC.$$

Bolometric magnitudes

$$M_{bol} = M_V + BC.$$

Class	Main Sequence	Giants	Supergiants
O3	-4.3	-4.2	-4.0
B0	-3.00	-2.9	-2.7
A0	-0.15	-0.24	-0.3
F0	-0.01	0.01	0.14
G0	-0.10	-0.13	-0.1
K0	-0.24	-0.42	-0.38
M0	-1.21	-1.28	-1.3
M8	-4.0		

Stellar luminosities

Total energy output of a star per unit time is known as luminosity. Bolometric magnitudes and luminosities are related by Pogson's equation

$$M_{\text{bol(sun)}} - M_{\text{bol}} = 2.5 \log (L/L_{\text{(sun)}})$$

Using Stefan's law, for a star of radius R and the area of the Sphere $4\pi R^2$

$$L = 4\pi R^2 \sigma T_e^4$$

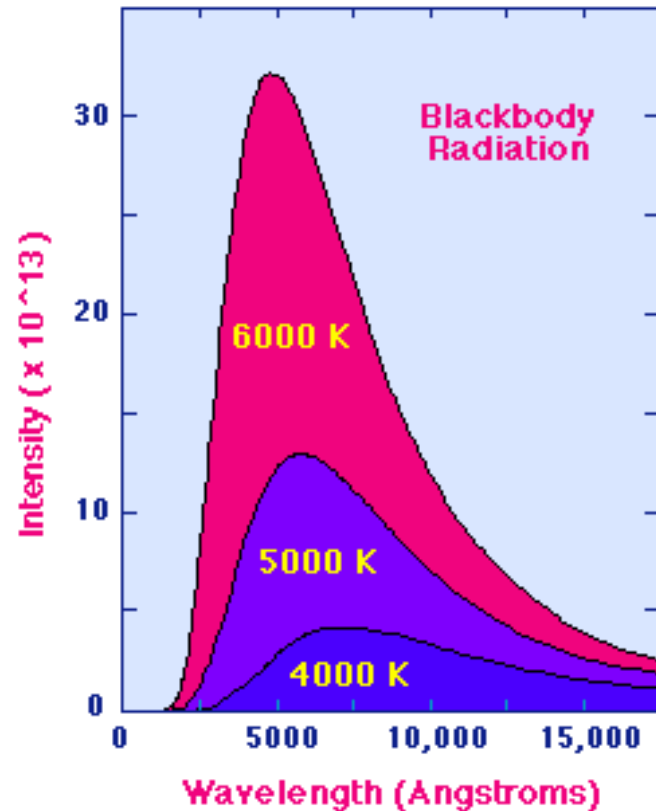
Say the star is at a distance r from the observer. Energy received on the earth per unit area is

$$E = L / 4\pi r^2 = \frac{1}{4} \theta^2 \sigma T_e^4$$

Where θ is the angular diameter of a star. And in the case of Sun this is written as

$$E_{\text{sun}} = \frac{1}{4} \theta_{\text{sun}}^2 \sigma T_{e\text{sun}}^4; \quad \theta = 1920'' \text{ for the Sun}$$

Black body radiation:



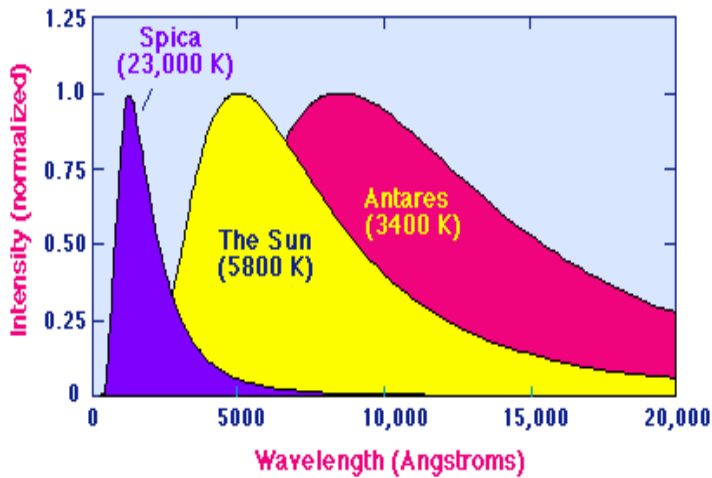
$$E(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

$$h = 6.625 \times 10^{-27} \text{ erg-sec (Planck Constant)}$$

$$k = 1.38 \times 10^{-16} \text{ erg/K (Boltzmann Constant)}$$

$$c = 3 \times 10^{10} \text{ cm/sec (Speed of Light)}$$

The Wien and Stefan-Boltzman law



Steffan - Boltzmann Law:

$$E = \sigma T^4$$

$$\sigma = 5.6705 \times 10^{-5} \text{ erg} \cdot \text{cm}^2 \cdot \text{K}^{-4} \cdot \text{sec}^{-1}$$

(Steffan - Boltzmann Constant)

Wien Displacement Law:

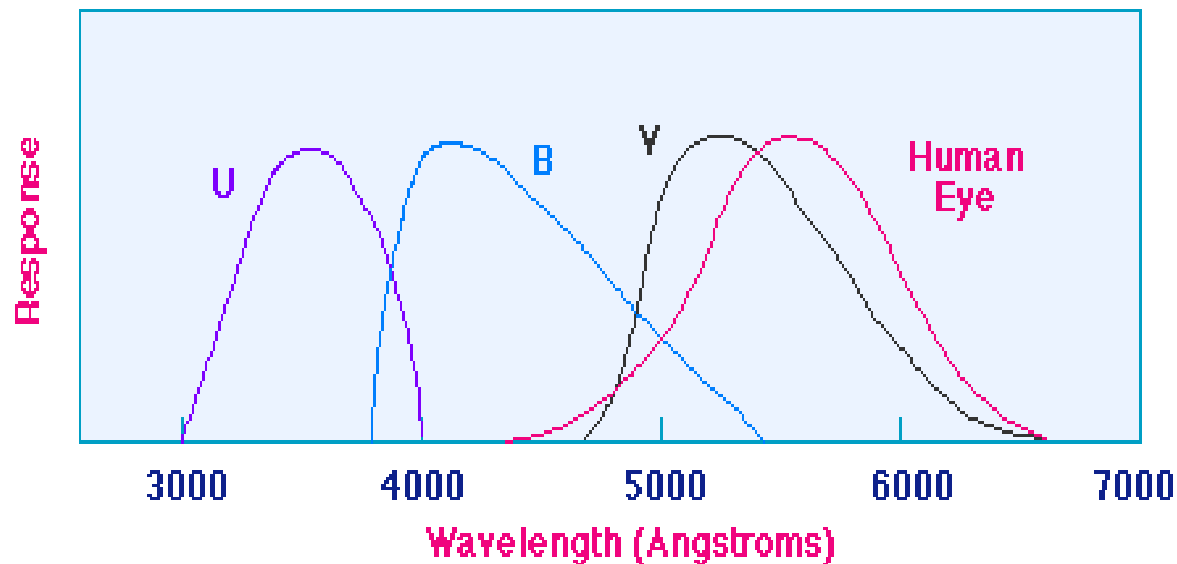
$$\lambda_{\text{Max}} = \frac{3 \times 10^7}{T}$$

(λ in Angstroms, T in Kelvin)

The Wien Law gives the wavelength of the peak of the radiation distribution, while the Stefan-Boltzmann Law gives the total energy being emitted at all wavelengths by the blackbody (which is the area under the Planck Law curve). Thus, the Wien Law explains the shift of the peak to shorter wavelengths as the temperature increases, while the Stefan-Boltzmann Law explains the growth in the height of the curve as the temperature increases. Notice that this growth is very abrupt, since it varies as the fourth power of the temperature.

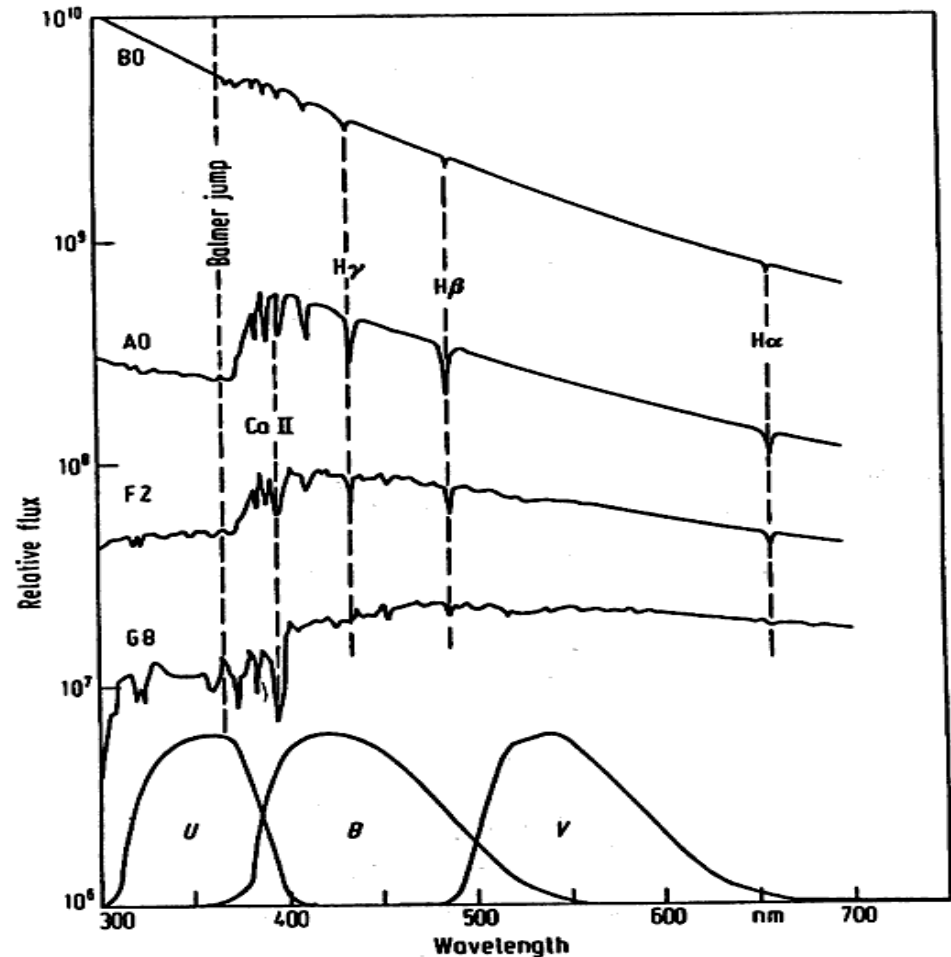
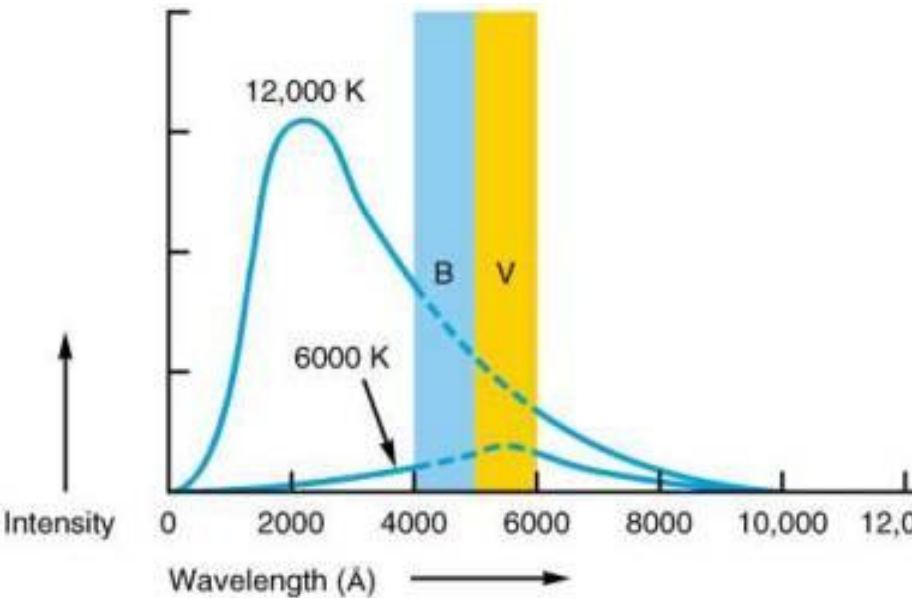
Filters

Optical devices called *filters* may be devised that allow light to pass in a limited range of wavelengths. In astronomy, a variety of filters are used to emphasize light in a particular wavelength region, but the most common are called the U (ultraviolet), B (blue), and photovisual (V) filters. Their transmission of light as a function of wavelength, as well as the response of the average human eye, is illustrated in the following figure. **This is known as broadband UBV Or Johnson photometry.**



Filters: Colour Indices

A *color index* is defined by taking the difference in magnitudes at two different wavelengths. For example, the B-V color index is defined by taking the difference between the magnitudes in the blue and visual regions of the spectrum and the U-B color index is the analogous difference between the UV and blue regions of the spectrum.



$$\text{Colour Index} = m_B - m_V = B - V$$

Filters: Colour Indices

The star Spica has apparent magnitudes $U = -0.24$, $B = 0.7$, and $V = 0.9$ in the UV, blue, and visual regions, respectively. Its colours are

$$\mathbf{B - V = 0.7 - 0.9 = - 0.2}$$

$$\mathbf{U - B = -0.24 - 0.7 = - 0.94}$$

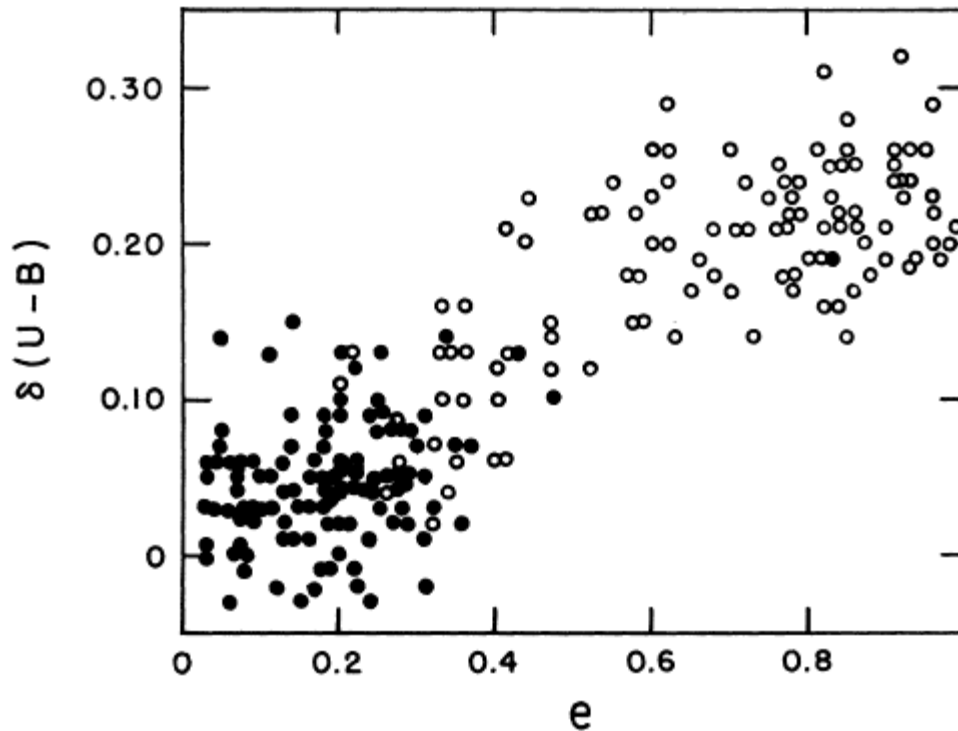
Negative values of color indices indicate that Spica is a hot star, with most of its radiation coming at shorter wavelengths. On the other hand, for Antares $B = 2.7$ and $V=0.9$, and the $B - V$ color index is

$$\mathbf{B - V = 2.7 - 0.9 = 1.8}$$

The positive value indicates that Antares is a cool star, with most of its radiation coming at longer wavelengths

UV excess: Formation of the Galaxy

“The stars with largest UV excess are moving in highly elliptical orbits, where as stars with little or no excess move in circular orbits”



Rapid collapse model for the Galaxy formation

Eggen, Lynden-Bell and Sandage (1962)

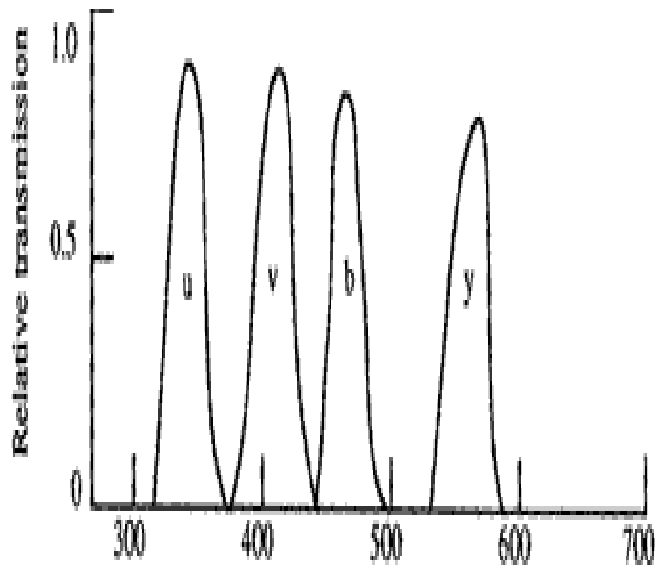
FIG. 4.—The correlation between the ultraviolet excess, $\delta(U - B)$, and the orbital eccentricity, e , for our sample of 221 stars. The filled and open circles represent stars from our first and second catalogues, respectively.

Filters: Stromgren four colour system

The Stromgren system is an intermediate-band width system which provides astrophysically important information.

y (yellow) filter matches the visual magnitude and corresponds to V

b (blue) filter is centered about 300Å to the red of B filter in UBV to reduce the effects of line blanketing.



Filter	Peak	bandpass
u	3500 Å	340 Å
v	4100 Å	200 Å
b	4700 Å	160 Å
y	5500 Å	240 Å
β	4860 Å	30 Å, 150 Å

Sromgren System:

(b-y) is a good indicator of color and effective temperature

In the absence of blanketing, the continuum slope would be roughly equal to (v-b).

v filter is defined such a way (v-b) is affected by blanketing and the difference,

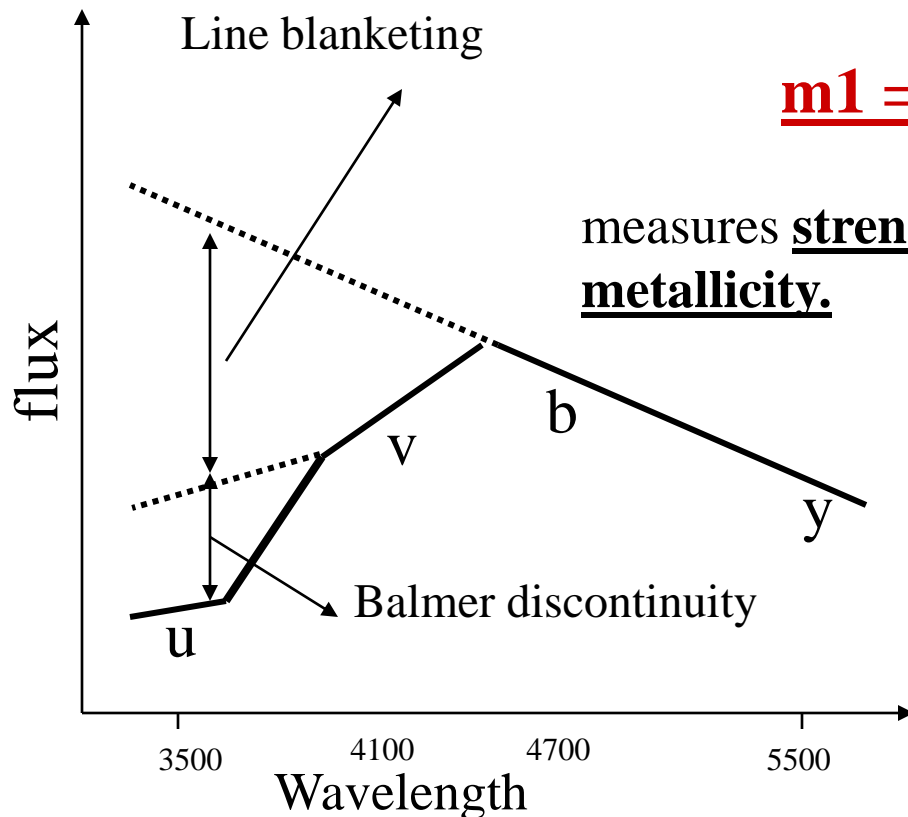
$$\underline{m1 = (v-b) - (b-y)}$$

measures strength of the blanketing and hence the metallicity.

The Balmer discontinuity, the index c1 is defined as

$$\underline{c1 = (u-v) - (v-b)}$$

nearly free from line blanketing.



uvby photometry: large scale Galactic surveys

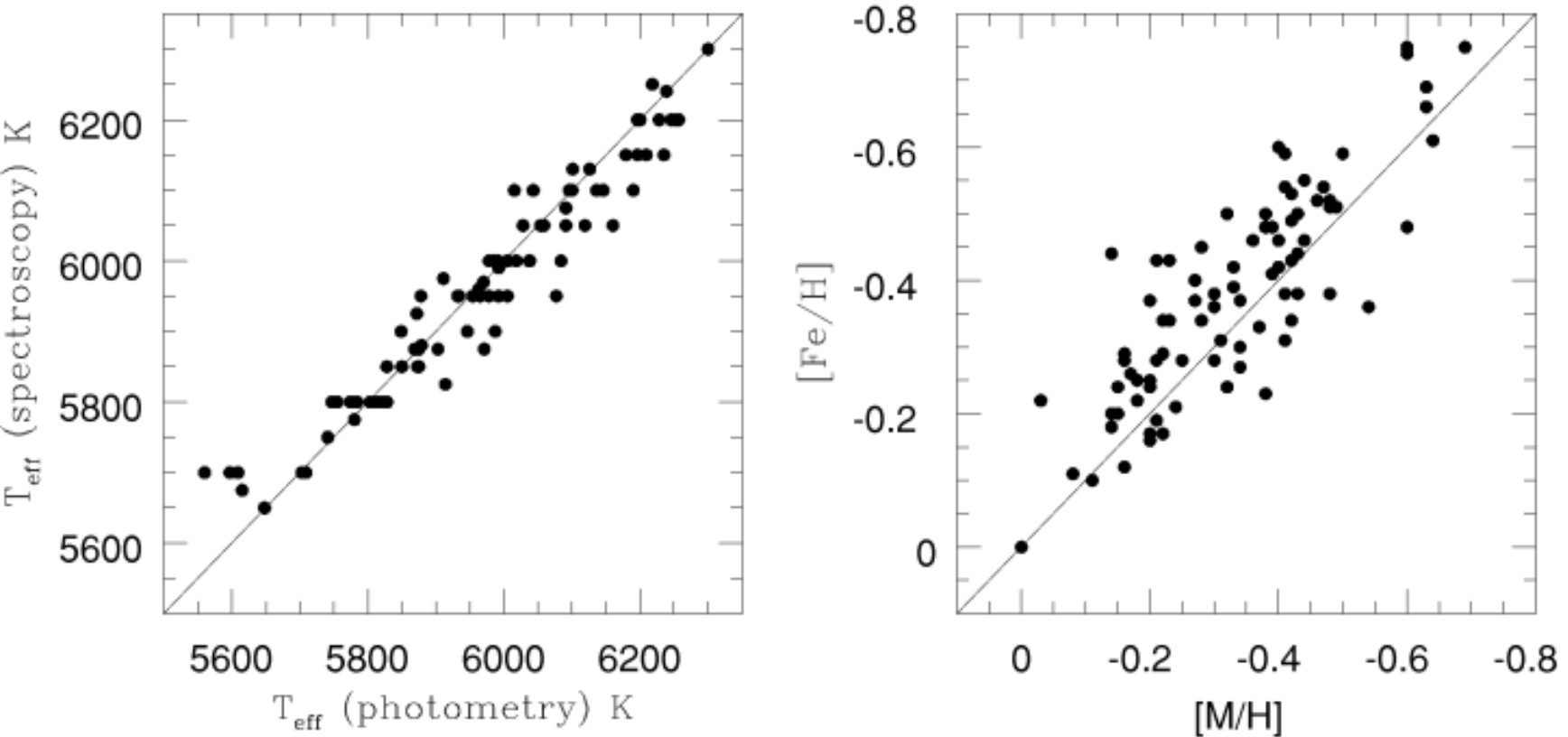


Fig 2. Photometric T_{eff} and $[M/H]$ for a sample of 82 stars compared with our (Reddy et al. 2003) spectroscopically derived T_{eff} and $[Fe/H]$ values.

$$\begin{aligned} \text{Difference } T_{\text{effs}} - T_{\text{eff}}(p) &= 71 \pm 47 \text{ K} \\ [Fe/H] - [M/H] &= 0.05 \pm 0.09 \end{aligned}$$

Magnitude calibration

The observed flux (f) is related to the actual stellar flux (f_{λ}^0) outside the earth's atmosphere, by

$$f = \int_0^{\infty} f_{\lambda}^0 T_{\lambda} R_{\lambda} S_{\lambda} d\lambda$$

Actual stellar flux

Transmission of atmosphere

Efficiency of telescope+detector

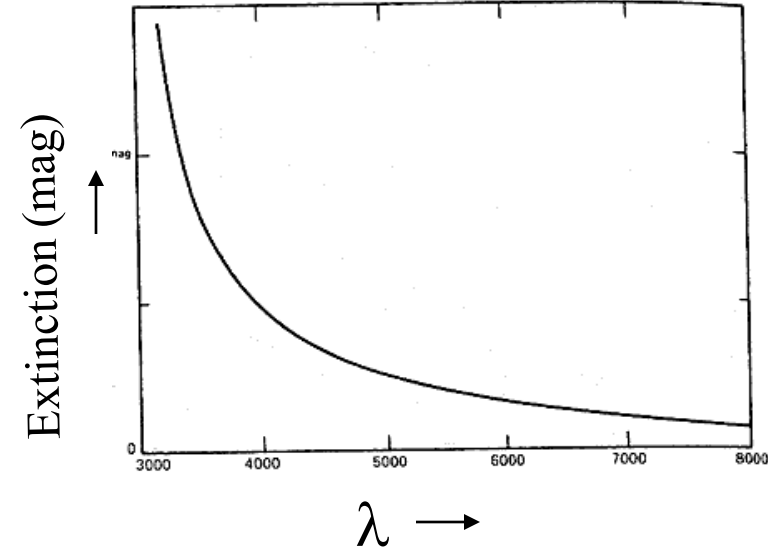
Transmission of the filter

Some of the material is adopted from the source:

<http://www.astro.virginia.edu/class/majewski/astr313/>

Magnitude Calibration: Atmospheric extinction

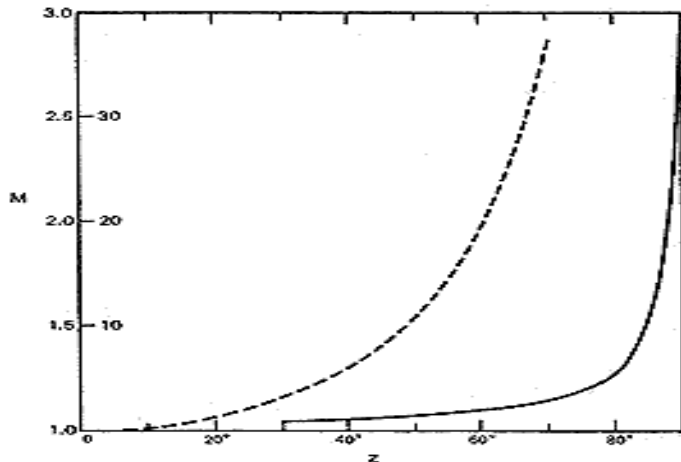
Extinction in mag. as a function of λ at zenith



- At $z=60^\circ$ you look through 2 airmass (from plane parallel approximation).

- At $z=71^\circ$ you look through 3 airmass (from spherical shell atmosphere model).

Airmass (mag) as a function of z



- At limit of $z=90^\circ$ you look through 38 airmass (derived with spherical shell formula).

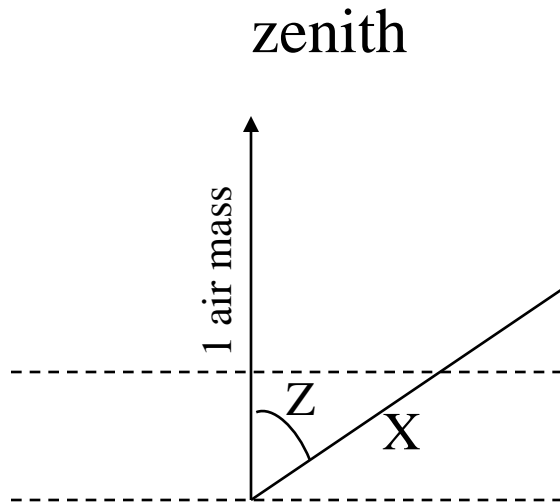
Magnitude calibration: Atmospheric extinction

To calculate proper magnitudes on an absolute scale, one needs to correct for extinction in photometry

quote mags as seen at "top of atmosphere"

Need to understand how many magnitudes of flux lost per given amount of atmosphere

depends on altitude above horizon



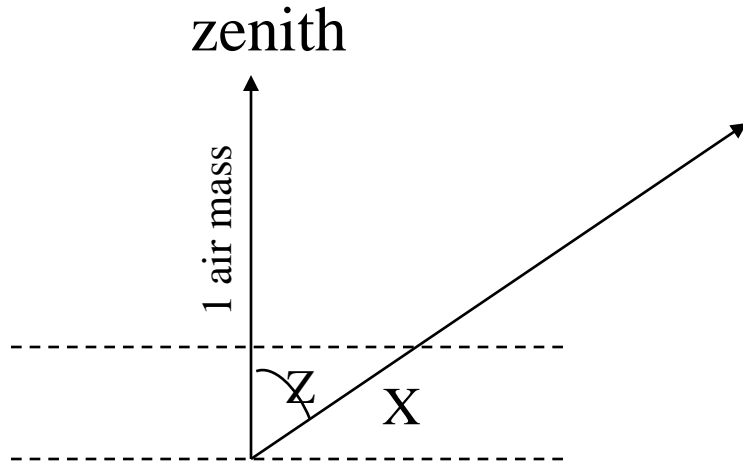
Z: zenith distance angle between zenith and the star:
(90° - altitude)

One airmass: amount of atmosphere seen at $Z = 0^\circ$

X : total atmosphere column determined in units of airmass

Atmospheric extinction

Using plane parallel approximation

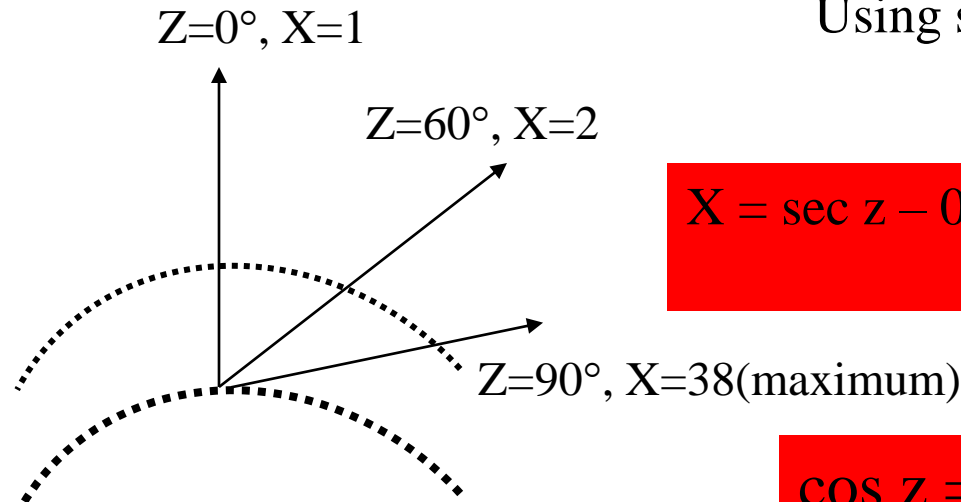


$$\cos z = 1/X$$

or

$$X = \sec z \quad (\text{good for } Z \leq 60^\circ)$$

Using spherical shell atmospheric correction



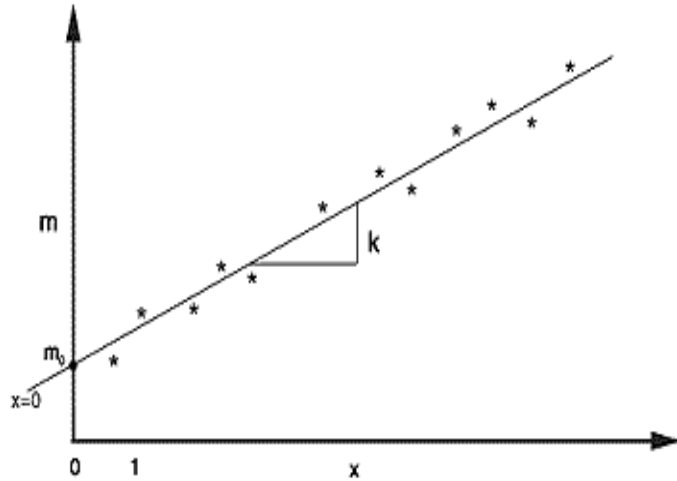
$$X = \sec z - 0.0018167 (\sec z - 1) - 0.002875 (\sec z - 1)^2 - 0.0008083 (\sec z - 1)^3$$

or

$$\cos z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos h$$

φ : your latitude; δ : star's dec; h : hour angle

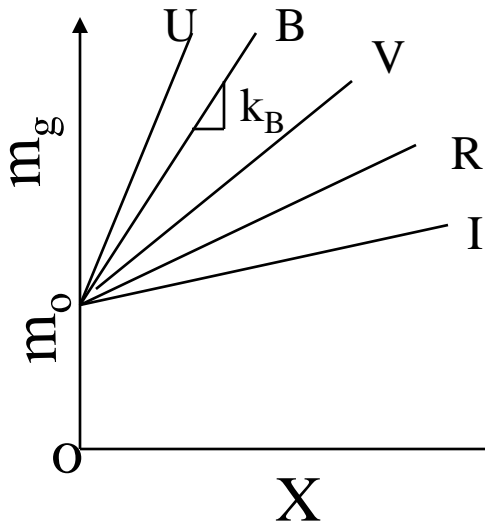
Atmospheric extinction



The linear relationship between loss of brightness in magnitudes and airmass is known as **Bouguer's law**.

The constant of proportionality, k , is called the **extinction coefficient**.

To solve for k we need to monitor a star as it changes its airmass (position with respect to the zenith) and apparent brightness (in magnitudes).



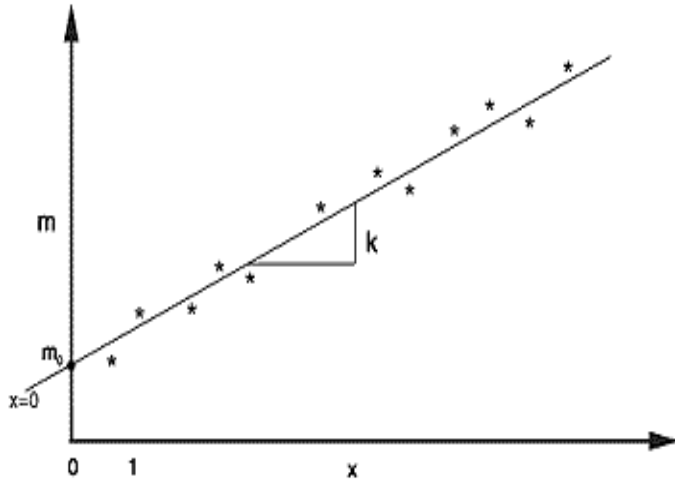
We know **k is $k(\lambda)$**

Thus,

$$V_{(X=0)} = V_X - k_V X$$

$$B_{(X=0)} = B_X - k_B X \text{ and so on ...}$$

Atmospheric extinction: correction



k is steeper as we go from red to blue: for standard UBVRI filters k is approximately

$$k_U = 0.50 \text{ mag/X}$$

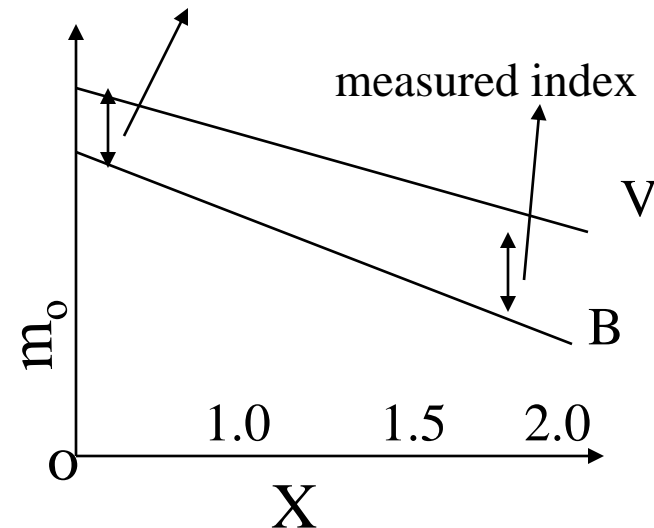
$$k_B = 0.25 \text{ mag/X}$$

$$k_V = 0.20 \text{ mag/X}$$

$$k_B = 0.25 \text{ mag/X}$$

$$k_I = 0.05 \text{ mag/X}$$

Corrected colour index

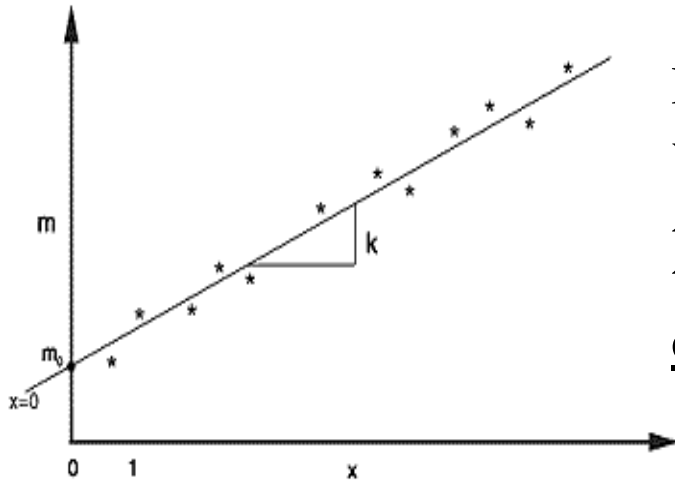


color shows an airmass effect. The colour extinction coefficient is written as

$$\begin{aligned} C_0 &= B_0 - V_0 = (B_X - k_B X) - (V_X - k_V X) \\ &= (B - V)_X - X(k_B - k_V) \\ &= C - k_c X \end{aligned}$$

higher the airmass redder the stars

Atmospheric extinction: first-order correction



Extinction is difficult model as it depends many A variables within the atmosphere. A first order Approximation is to account for the largest contributor, the air mass variation. The **first order extinction correction terms** are written as follows

$$\begin{aligned}v_0 &= v - k'_v X \\(b-v)_0 &= (b-v) - k'_{bv} X \\(u-b)_0 &= (u-b) - k'_{ub} X\end{aligned}$$

Where, k' is called principalexinctioncoefficient (magnitudes per unit airmass) and the subscript 0 denotes magnitudes above the atmosphere.

The values of the extinction coefficients can then be found by following One star thru changing air masses and plotting the color index or magnitude Versus X. The slope of the line is the k' and the intercept is m_0

Magnitude calibration: reduction to a standard system

In addition to correcting for airmass, one also needs to account for any differences between your equipment (telescope + detector + filter) and the standard system of equipment (i.e. standard bandpass).

Standard magnitudes and colors are written as

$$\begin{aligned} V &= \varepsilon(B-V) + v_0 + \zeta_v \\ (B-V) &= \mu(b-v)_0 + \zeta_{bv} \\ (U-B) &= \psi(u-b)_0 + \zeta_{ub} \end{aligned}$$

U, B, V are standard magnitudes, ζ 's are zero-points. ε , μ , ψ are called Transformation coefficients. Values with 0 subscript are the values corrected for atmospheric extinction

Zero-points and transformation coefficients could be determined by measuring several stars whose standard magnitudes and colors are known. The slope of the best-fitted line for a plot of $(V-v_0)$ versus $(B-V)$ will be the ε .

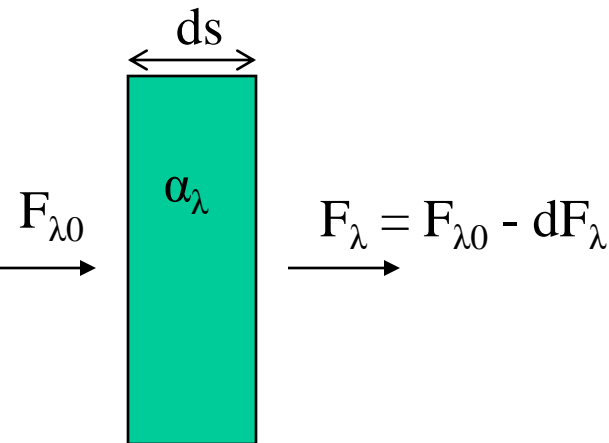
(For more details and examples please see Henden and Kaitchuk)

Extinction correction derivation:

Ignoring curvature of the earth and assuming plane parallel atmosphere
And altitude above 30° , one can write relation between the flux F_{λ_0}
Above the atmosphere and F_λ thru the atmosphere,

$$(1) F_\lambda/F_{\lambda_0} = \exp(-\int_0^s \alpha_\lambda ds); \quad \alpha_\lambda \text{ absorption coefficient}$$

$$(2) \tau_\lambda = \int \alpha_\lambda ds; \quad \text{is the optical depth}$$

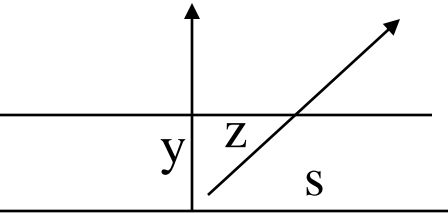


$$(3) \quad \text{Thus, } F_\lambda/F_{\lambda_0} = e^{-\tau_\lambda} \quad (1) + (2)$$

Converting these into magnitudes, one may write as

$$(4) \quad m_\lambda - m_{\lambda_0} = -2.5 \log F_\lambda/F_{\lambda_0}$$

Extinction correction derivation:



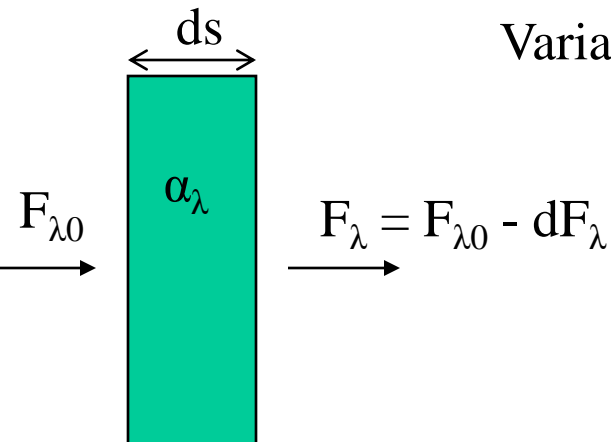
$$(5) \quad m_{\lambda} - m_{\lambda 0} = -2.5 \log (e^{-\tau_{\lambda}}) ; (3) + (4)$$

$$m_{\lambda} - m_{\lambda 0} = 2.5 (\log (e) \tau_{\lambda})$$

or

$$(6) \quad m_{\lambda 0} = m_{\lambda} - 1.086 \tau_{\lambda}$$

Variation of τ with the star's position can be deduced as follows



$$(7) \quad \cos z = y/s \text{ or } s = y \sec z$$

$$(8) \quad ds = dy \sec z$$

$$(9) \quad \tau_{\lambda} = \sec z \int \alpha_{\lambda} dy \quad (2) + (8)$$

$$(10) \quad m_{\lambda 0} = m_{\lambda} - k' \sec z \quad (6) + (9)$$

Atmospheric extinction: second-order correction

Bandwidth of filters, particularly UBV system, has effect on the extinction and the corresponding correction is called second-order extinction correction. Within the band pass flux at some wavelengths suffer more than at others. Blue wavelengths suffer more than red ones. In the case of hot stars one may underestimate extinction by adopting mean extinction and for the red stars one may overestimate extinction.

We may modify the k' as

$$k'_v \rightarrow k'_v + k''_v (b-v)$$

$$k'_{bv} \rightarrow k'_{bv} + k''_{bv} (b-v)$$

$$v_0 = v - k'_v X - k''_v (b-v) X$$

$$(b-v)_0 = (b-v) - k'_{bv} X - k''_{bv} (b-v) X$$

To solve for k'' 's one needs to observe a close pair of stars with very different colours so that their air mass is almost same.

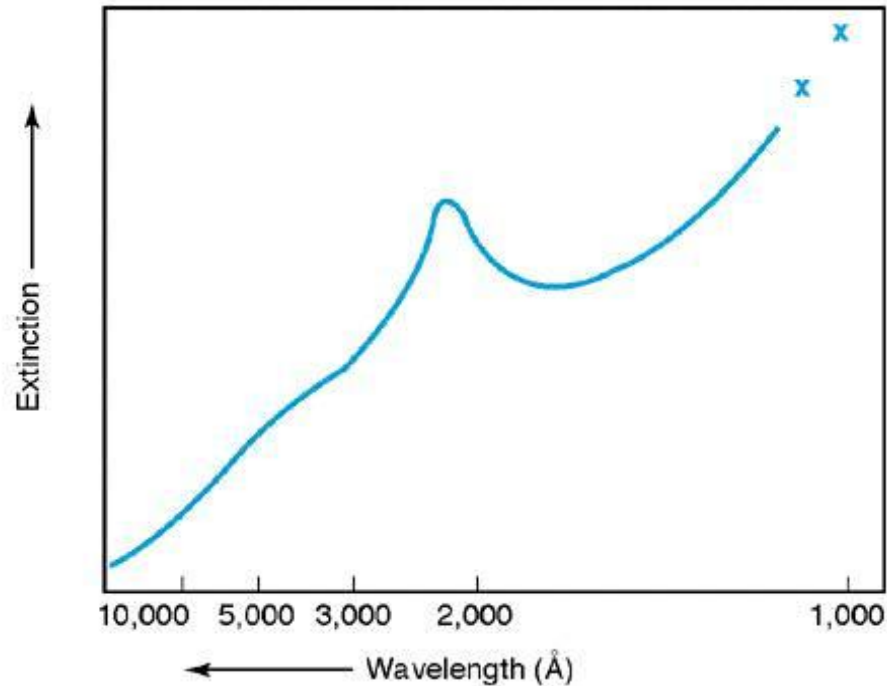
$$v_{01} - v_{02} = (v_1 - k'_v X - k''_v (b-v)_1 X) - (v_2 - k'_v X - k''_v (b-v)_2 X)$$

$$\Delta v_0 = \Delta v - k''_v \Delta(b-v) X$$

$$\text{Similarly, } \Delta(b-v)_0 = \Delta(b-v) - k''_{bv} \Delta(b-v) X$$

Interstellar Extinction

The amount of extinction varies as a function of wavelength such that it is highest at short wavelengths and lowest at longer wavelengths. There are approximately 30 magnitudes of visual extinction towards the centre of our galaxy. Such regions of high extinction are investigated at infrared and radio wavelengths for obvious reasons!



Note the 2200 Angstrom bump possibly due to the presence of carbon in the form of spherical graphite grains in the ISM.

Interstellar Extinction

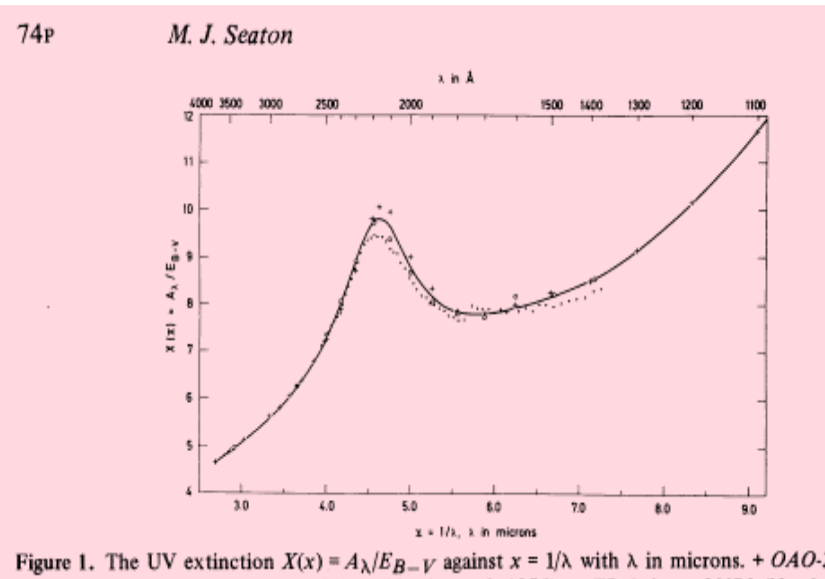
The observed colour index $B-V$ is related to the intrinsic colour index $(B-V)_0$:

$$B-V = (B-V)_0 + E(B-V)$$

Where $E(B-V)$ is the **colour excess**

For normal regions of the ISM the colour excess is related to the visual extinction A_V :

$$\frac{A_V}{E(B-V)} = 3.2 \pm 0.2$$



Values of $X(x) = A_\lambda/E_{B-V}$ for $1.0 \leq x \leq 2.7$

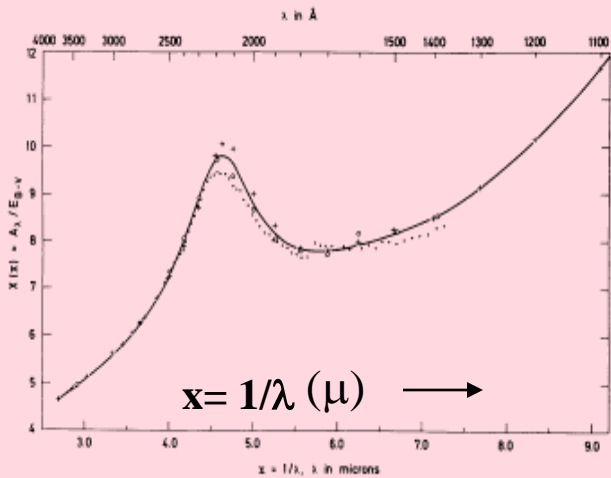
Interstellar Extinction

$$\frac{A_V}{E(B-V)} = 3.2 \pm 0.2$$

74p

M. J. Seaton

$$X(x) = A_\lambda / E_{B-V}$$



Values of $X(x) = A_\lambda / E_{B-V}$ for $1.0 \leq x \leq 2.7$

(adopted from Seaton 1979 MNRAS)

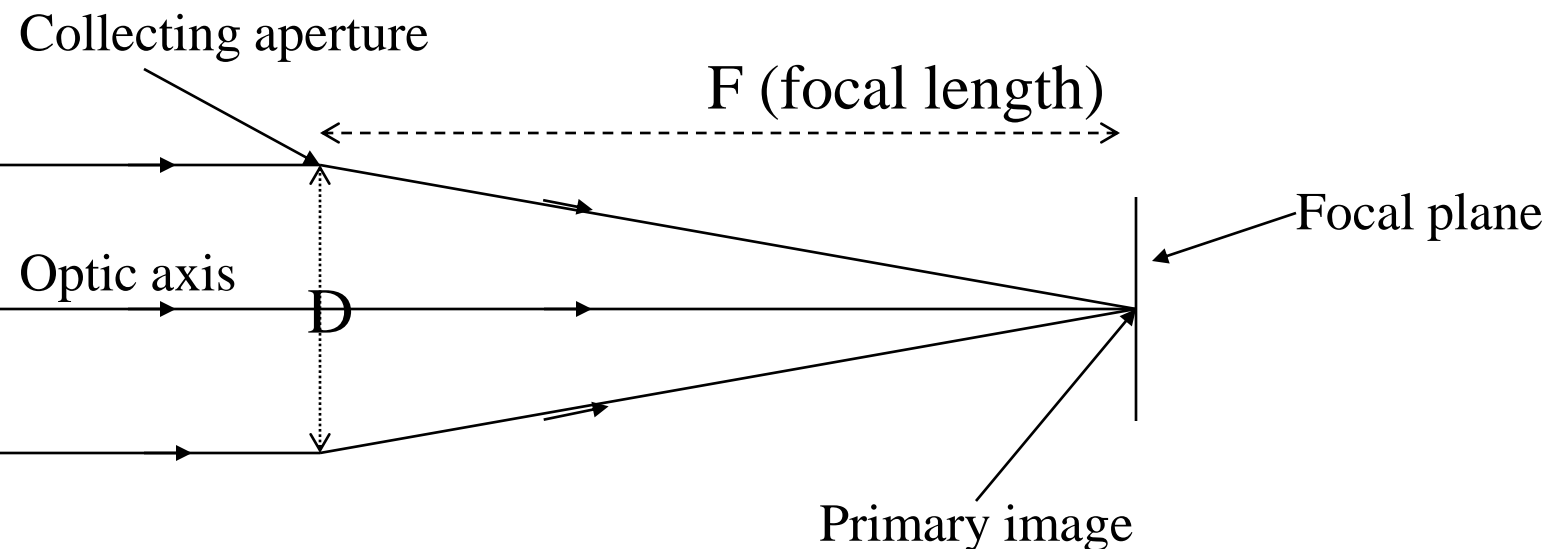
x	X(x)	x	X(x)
1.0	1.36	2.0	3.56
1.2	1.84	2.2	3.96
1.4	2.24	2.4	4.26
1.6	2.66	2.6	4.52
1.8	3.14	2.7	4.64

This helps to find out interstellar extinction at any given wavelength for the known values of $E(B-V)$.

Telescopes: A Few definitions

Two main functions of telescopes

1. To allow collection of photons over a larger area. Helps to detect fainter objects and to measure with greater accuracy
2. To allow higher angular resolution. Helps to resolve and study spatial information of extended objects.



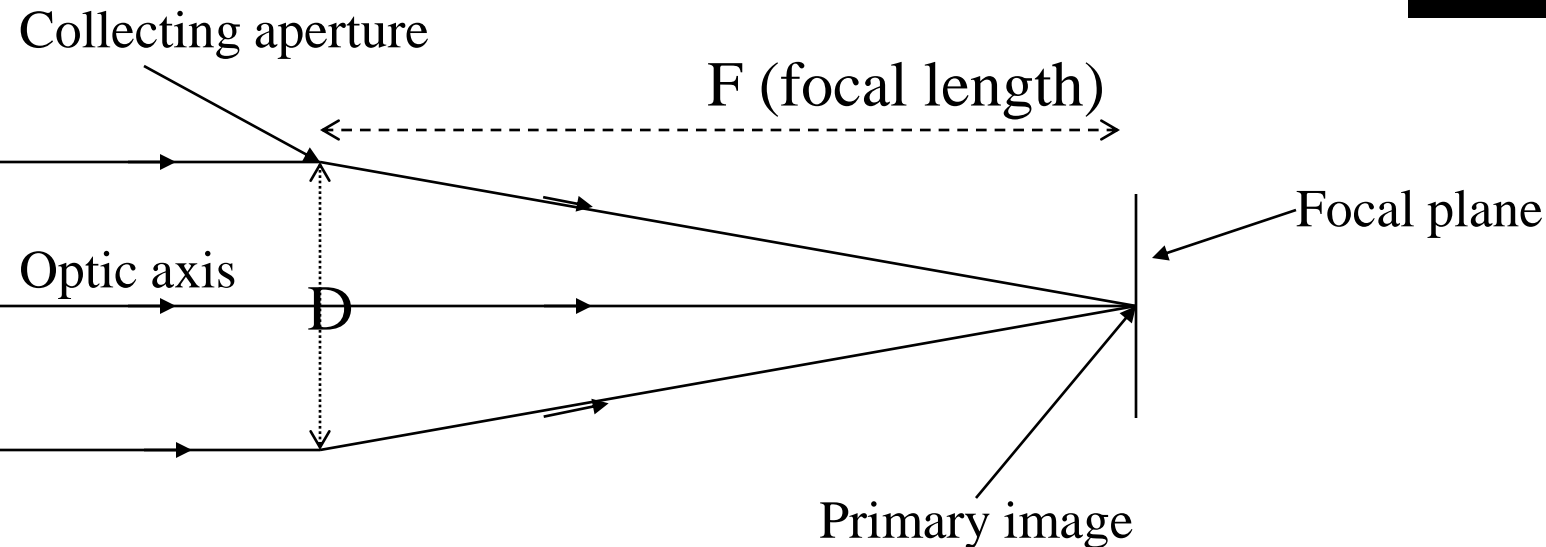
Telescopes: focal ratio (f)

Focal length (F) is the distance between the light collecting aperture and the primary image or the focal plane.

A plane through focal point and at right angles to the optic axis is called the focal plane.

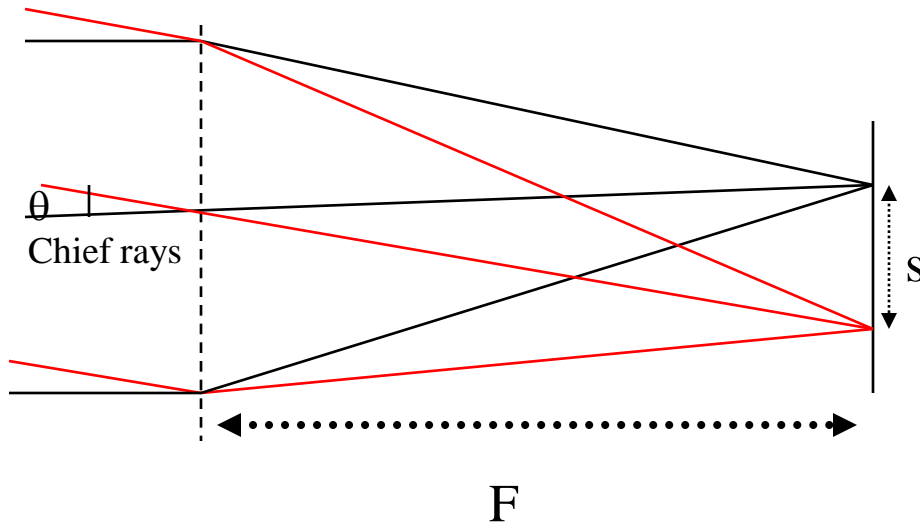
Focal ratio, f is defined as the ratio of the focal length F and the diameter D , of an aperture

$$f = F/D$$



Telescopes: Plate Scale

The relationship between the size of the image on the focal plane and the angular field in the sky represented by this image is governed by the focal length of the telescope.



θ is the angle between the rays coming from two stars in the sky and “s” is the linear separation between the two images in the focal plane,

$$\text{Thus, } s = F \tan \theta$$

$$\text{Or } s = F \theta \text{ as } \theta \text{ is very small}$$

θ is in radians. In order to study images in detail larger separation is required and hence long telescope focus. The correspondence between θ and s is called plate scale of the telescope and is expressed in arc-sec/mm

$$d\theta/ds = 1/F \text{ radians}$$

$$= 206265/F \text{ arc-sec/mm; } F \text{ and } s \text{ are in mm and } \theta \text{ is arc-sec}$$

The telescope: flux collector

If an object such as a star is considered as a point source, its telescope image is also considered as a point source, no matter how large a telescope is

The larger the telescope, the greater is the amount of collected radiation for detection

Stellar brightness, flux F , might be expressed in units of energy $\text{s}^{-1} \text{m}^{-2} \lambda^{-1}$

Determination of the signal-to-noise ratio of any measurement is performed in terms of photons arriving at the detector within some wavelength interval over a certain amount of time.

Number of photons at the telescope aperture

$$N = \pi/4 D^2 \times \Delta t \int_{\lambda_1} \lambda F_{\lambda} / hc d\lambda$$

energy associated with each photon $E = hc/\lambda$ is substituted

The telescope: signal-to-noise ratio (S/N)

Number of photons at the telescope aperture

$$N = \pi/4 D^2 \int_{\lambda_1}^{\lambda_2} \lambda F_\lambda / hc d\lambda \Delta t$$

D telescope diameter, Δt is integration time, λ_1 and λ_2 are the cut-on and cut-off wavelengths. For relatively constant photon energy over of the given wavelength One may write this as

$$N = \pi/4 D^2 \int_{\lambda_1}^{\lambda_2} \lambda F_\lambda / hc \Delta \lambda \Delta t$$

The arrival of photons at the telescope is a statistical process. When the arriving Flux is low, fluctuations are clearly seen which are said be **photon shot noise**

Uncertainty of any measurement is given by \sqrt{N} (square root of N)

This error = $N \pm \sqrt{N}$

$$S/N = N / \sqrt{N} = \sqrt{N} \propto \sqrt{D^2 \Delta t} \propto D \sqrt{\Delta t}$$

The telescope: resolving power

The resolving power of a telescope may be defined as the ability of a telescope to separate objects with a small angle between them. There is a fundamental limit to any telescope to resolve objects which is called the theoretical resolving power.

the optics of telescope collectors

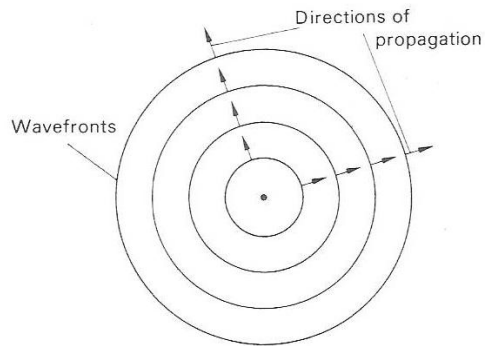


Figure 16.4. Wavefronts radiating from a point source.

Consider a point source and its radiation as wavefronts which are in phase.

As time proceeds wavefronts expand and their direction of propagation is right angles to the surface of wavefronts.

A star at infinity, the wavefronts are in the form of parallel Planes by the time they arrive at the telescope.

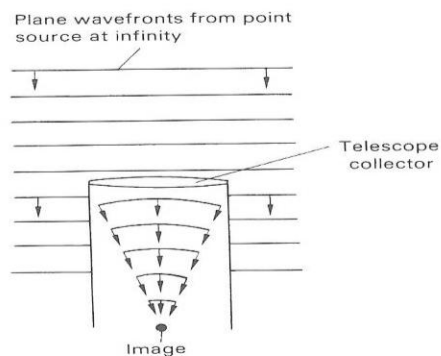


Figure 16.5. Wavefronts arriving at the telescope aperture

Effect of the aperture is to alter the shape of the wavefronts by introducing a differential phase change so that image is produced. It is in the form of a **diffraction pattern**.

The telescope: resolving power

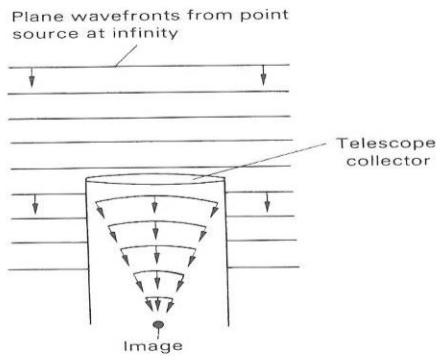
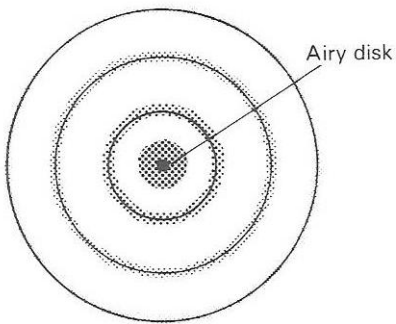


Figure 16.5. Wavefronts arriving at the telescope aperture

The pattern is caused by the interruption of the plane Wavefronts by the telescope aperture

Resultant diffraction pattern produced from a point object appears as a spot at the center of a system of concentric rings.

Energy contained in each ring decreases according to the number of the ring. It is predicted central spot contains 84% of the energy.



the diffraction pattern of a star image in the focal plane of a

The central spot is sometimes referred to as the Airy disk after it's first investigator.

The telescope: resolving power

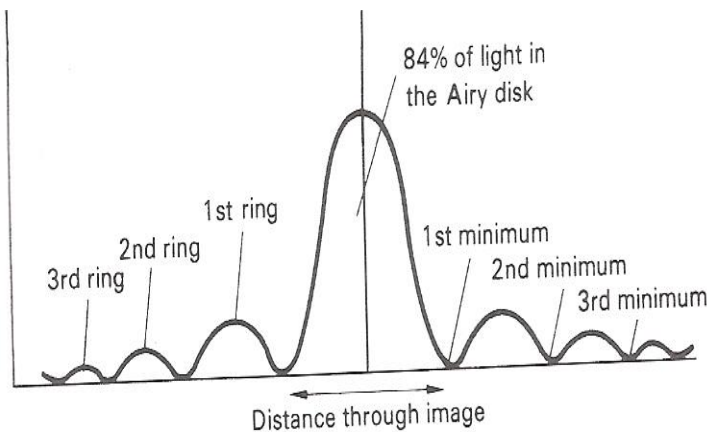


Figure 16.7. Intensity scan through a diffraction pattern.

Intensity scan along a line through the center of the pattern. The central spot shows maximum of intensity.

α is the angle subtended at the aperture by the center of the Airy disk and a point in the diffraction pattern.

Positions for minima intensity are given by

$$\sin \alpha_n = m_n \lambda / D$$

for small angle

$$\alpha_n = m_n \lambda / D$$

$$m = 1.22 \text{ for } n=1$$

$$m = 2.23 \text{ for } n=2$$

$$m = 3.24 \text{ for } n=3$$

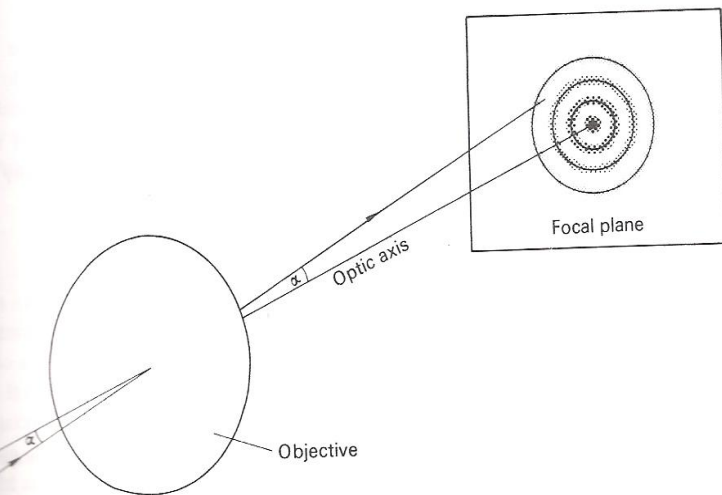


Figure 16.8. The diffraction pattern in the focal plane of the objective.

n = number of the minima, m is the numerical factor, λ is wavelength and D is the diameter of the telescope.

The telescope: resolving power

$$\alpha_n = m_n \lambda/D$$

It will only be possible to resolve resultant image as being two components if the individual Airy disks are well separated

According to Rayleigh's criterion, two images are said to be resolved when the center of one Airy disk falls on the first minimum of the other diffraction pattern.

Thus, it should be possible to resolve two stars if they are separate by an angle greater than

$$\alpha = 1.22 \lambda/D \text{ (radians)}$$

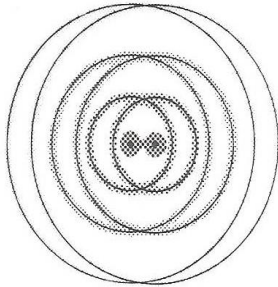
or

$$\alpha = 206265 \times 1.22 \lambda/D \text{ (arc-sec)}$$

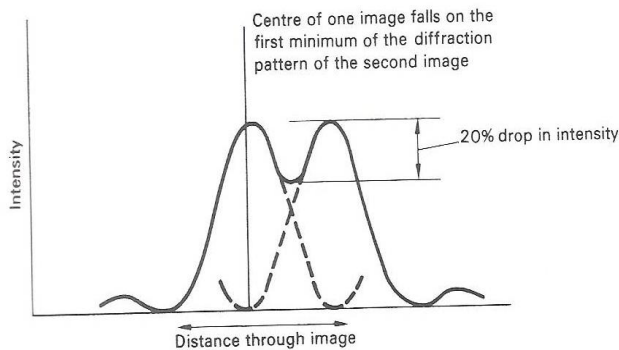
At 5500Å and for 2m telescope α is 0.07"

This is known as theoretical angular resolving power of the telescope

The optics of telescope collectors

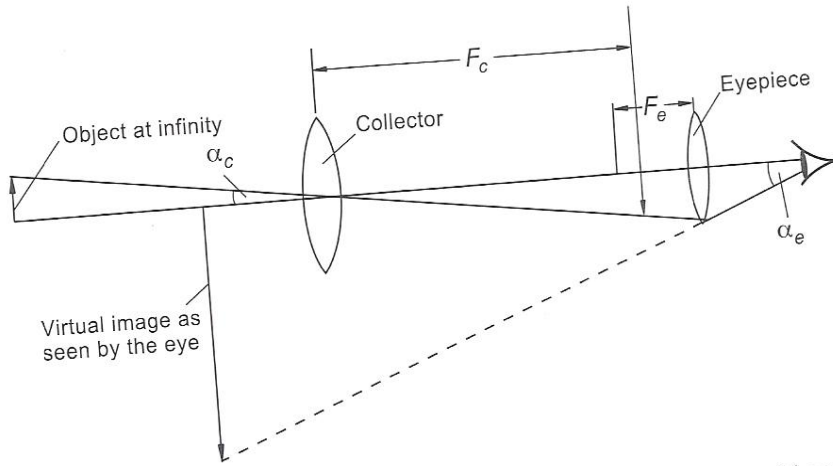


9. The combined diffraction pattern obtained from two point sources which are separated by Rayleigh's criterion.



10. Intensity scan through a diffraction pattern obtained from two point sources which are given by Rayleigh's criterion.

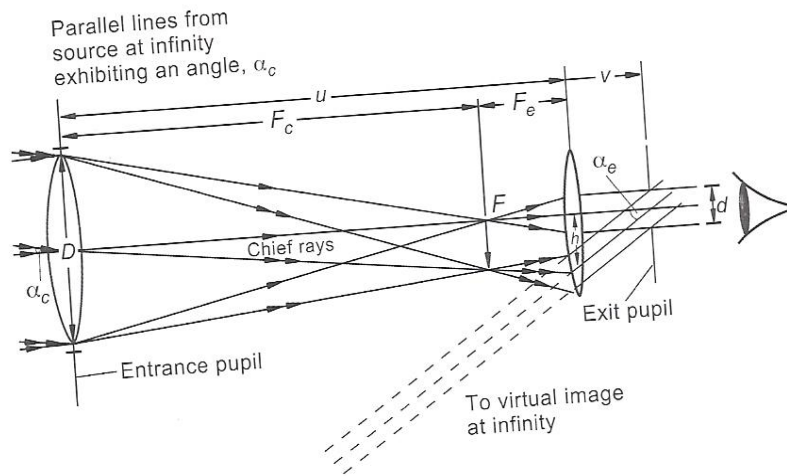
The telescope: magnifying power



The **magnifying power, m** , of the optical system is defined as the ratio of the angle subtended by the virtual image at the eye, α_e , and the angle α_c , subtended by the object at the aperture. Thus,

$$m = \alpha_e / \alpha_c$$

Figure 17.1. Visual use of a telescope. The collector is depicted as an objective; it could equally well be an eyepiece in a Galilei telescope system.



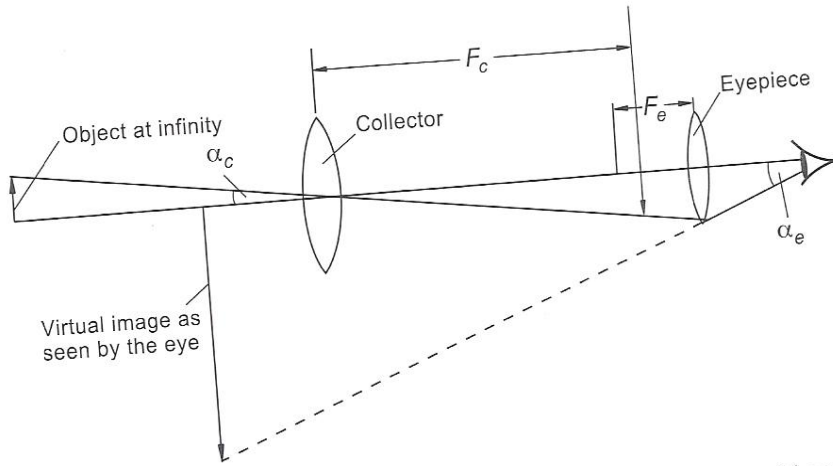
The aperture acts as the **entrance pupil** and the image of the aperture formed by the eyepiece acts as the **exit pupil**.

The distance from the eyepiece to the exit pupil is called **eye relief (v)**

Figure 17.2. A schematic diagram of the astronomical telescope, illustrating the positions of the entrance and exit pupils.

All the rays from the field which can be viewed by the telescope pass thru the exit pupil and eye should be placed at this position

The telescope: magnifying power



7.1. Visual use of a telescope. The collector is depicted as an objective; it could equally be a system.

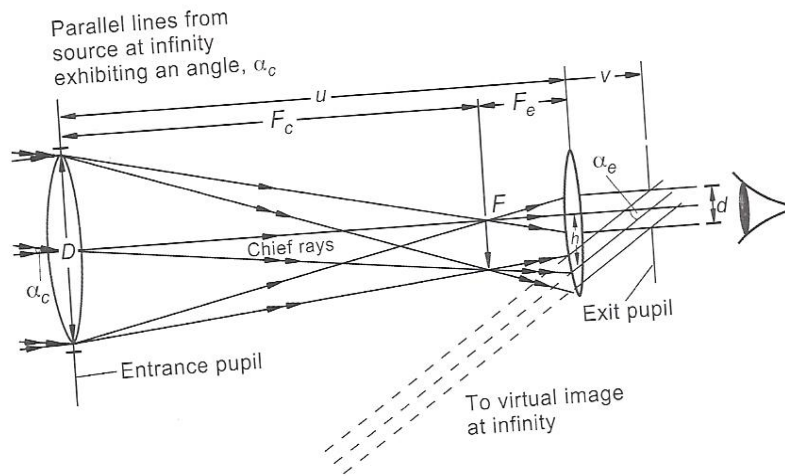


Figure 17.2. A schematic diagram of the astronomical telescope, illustrating the positions of the lenses.

$$m = \alpha_e / \alpha_c$$

From figure, we may write

$$\tan \alpha_e = h/v \quad \text{or} \quad \alpha_e = h/v$$

$$\tan \alpha_c = h/u \quad \text{or} \quad \alpha_c = h/u$$

Where, h distance from optical axis to chief ray at the eyepiece

Using general lens formula

$$1/u + 1/v = 1/F_e$$

We write,

$$1/v = u - F_e / u F_e \quad \text{and}$$

$$1/v = F_c / F_e (F_c + F_e)$$

Where $u = F_c + F_e$ (see the figure)

The telescope: magnifying power

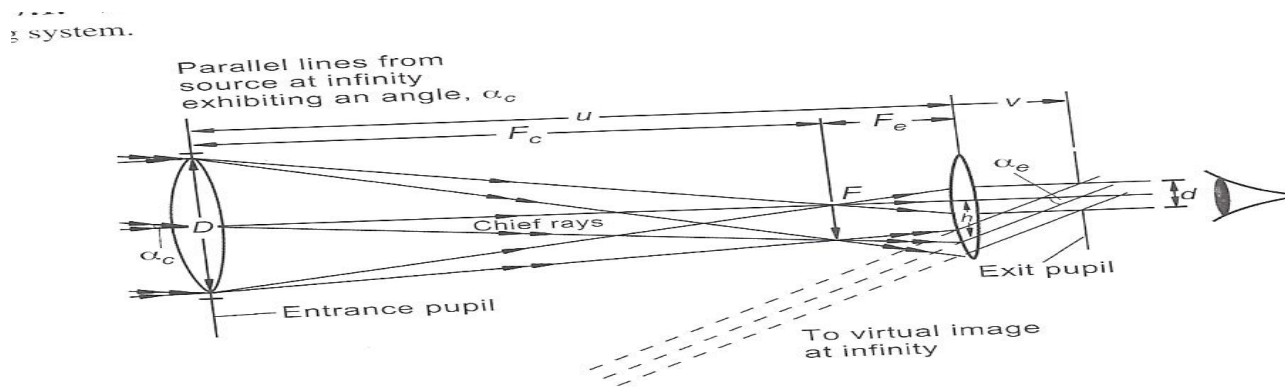


Figure 17.2. A schematic diagram of the astronomical telescope, illustrating the positions of the lenses.

$$\alpha_e = hF_c / F_e(F_c + F_e) \text{ and}$$

$$\alpha_c = h / F_c + F_e$$

$$\text{Therefore, } m = \alpha_e / \alpha_c = F_c / F_e$$

$$\text{Also } m = D / d$$

Many stars which appear to be single to the unaided eye are found to be double when viewed with the telescope due to magnification of the separation.

Where, D is the diameter of the aperture and d is the diameter of the exit pupil

The telescope: magnifying power

system.

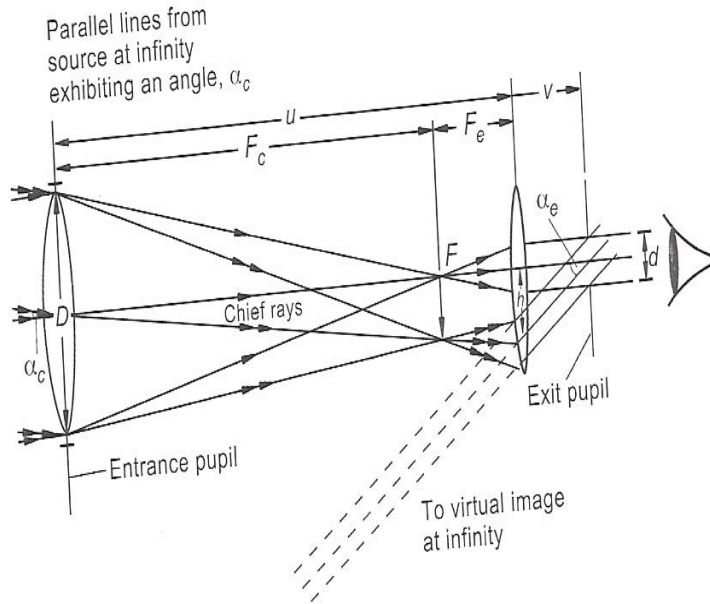


Figure 17.2. A schematic diagram of the astronomical telescope, illustrating the positions of the lenses.

Magnification limits:

lower limit: $m \geq D/d$ or $\geq D/8$

Where $d \sim 8\text{mm}$ for pupil of the eye (exit pupil \leq entrance pupil of the eye)

This suggests all the light collected at the aperture is available for viewing by the eye. For less m values, some light is lost.

The upper limit is set by the impracticality of making eyepieces with very small focal length. For the image to be seen without loss in the quality of the eye's Function, the exit pupil must be larger than 0.8mm. Thus

upper limit $m \leq D/0.8$

The telescope: field of view (FOV)

The field of view is a function of the optics of the eyepiece itself and its magnification which is a function of the telescope focal length. Typical eyepieces have field of view ranging from 40° to 65° or more..

There are three basic facts to know about any eyepiece, the *eyepiece focal length* (e), the diameter of the exit pupil, and the *eyepiece field of view* known as the effective field of view.

The *eyepiece field of view* is the theoretical field of view in degrees the eyepiece would provide at a *magnification of one*. The *eyepiece field of view* varies with the type of eyepiece. The actual *telescope field of view* is calculated by dividing the *eyepiece field of view* by the *magnification*.

$$\begin{aligned} \text{Eyepiece field of view} &= 40^\circ \\ \text{Telescope field of view} &= \text{Eyepiece field of view} / \text{Magnification} \\ &= 40/48 \\ &= 0.83 \text{ degrees} \\ &= 50.0 \text{ minutes of arc} \end{aligned}$$

The telescope: limiting magnitude

The amount of energy collected by an aperture is proportional to its area.

Amount of power, P (ergs/s) collected from flux, f (ergs s⁻¹ mm⁻²) depends on the area of the telescope aperture

$$P = fA \text{ or } P \propto fd^2$$

$$f \propto p/d^2$$

Using Pogson's equation, we form the relation

$$m = -2.5 \log_{10} f + c$$

$$m = -2.5 \log_{10} (p) + 5 \log_{10} (d) + \text{const.}$$

We define the limiting magnitude as the magnitude where the received power P drops to an arbitrarily low value below which eye or eye+telescope can't detect the source.

For star light, the limit of unaided eye detection is set at about 6th magnitude.

The telescope: limiting magnitude

Thus, $m_{lim} = -2.5 \log_{10} (P_{lim}) + 5 \log_{10} (d) + \text{const}$

All other things are equal (sky condition, distance to the source) the limiting magnitude of the telescope depends only on the diameter of the aperture

$$m_{lim1} = m_{lim2} + 5 \log_{10} (d1) - 5 \log_{10} (d2)$$

$d2 = 8\text{mm}$ for the diameter of the eye pupil and $m_{lim2} = 6$ for the unaided eye

$$\begin{aligned} m_{lim1} &= 6 + 5 \log_{10}(d1) - 5 \log_{10}(8) \\ m_{lim} &= 1.485 + 5 \log_{10}(d) \end{aligned}$$

For 50cm telescope, m_{lim} is about 15.0

(this is for a perfect telescope with 100% transmission efficiency)

By rounding off the figures, the equation for the m_{lim} may be written as

$$m_{lim} = 6 + 5 \log_{10}(D) - 5$$

For 50 cm telescope, m_{lim} is about 14.5.

D diameter of the telescope and is expressed in mm

The telescope: limiting magnitude ($\propto D t$)

A star's image on a photographic plate (CCD) can be considered to be a point. Energy collected into the point is proportional to the apparent brightness of the star, the area of the aperture, and integration time of the exposure.

The eye needs to receive about 200 photons/s to sense an image.

$$E_e = 200 \times \frac{4}{\pi} \times d^2 \text{ photons s}^{-1} \text{ mm}^{-2} \quad (1)$$

Where $d = 8\text{mm}$ for the dark adapted eye pupil

a photographic star image requires collection of 50000 photons to register.

Energy arrival rate E_t per unit area and per unit time at the telescope aperture is written as

$$E_t = \frac{50000}{tD^2} \times \frac{4}{\pi} \text{ photons s}^{-1} \text{ mm}^{-2} \quad (2)$$

Where, D is diameter of the telescope and is expressed in mm, t is the exposure time in seconds

The telescope: limiting magnitude ($\propto D t$)

$$E_e = 200 \times 4/\pi \times d^2 \text{ photons s}^{-1} \text{ mm}^{-2} \quad (1)$$

$$E_t = 50000/tD^2 \times 4/\pi \text{ photons s}^{-1} \text{ mm}^{-2} \quad (2)$$

Pogson's equation allows us to form the relation

$$m_t - m_e = -2.5 \log E_t/E_e$$

$$m_t - m_e = 2.5 \log_{10} (tD^2) - 2.5 \log_{10} (1.6 \times 10^4)$$

By putting $m_e = 6$ the value obtained for m_t corresponds to the limiting magnitude m_{lim} of the star which can be recorded by a telescope of diameter D and exposure time t (s), hence

$$\begin{aligned} m_{\text{lim}} &= 6 + 5 \log_{10} D + 2.5 \log_{10} t - 10.5 \\ &= -4.5 + 5 \log_{10} D + 2.5 \log_{10} t \end{aligned}$$

For 50cm telescope with an exposure time of 1000 s, one may record a star of 16.5 mag.

Telescopes

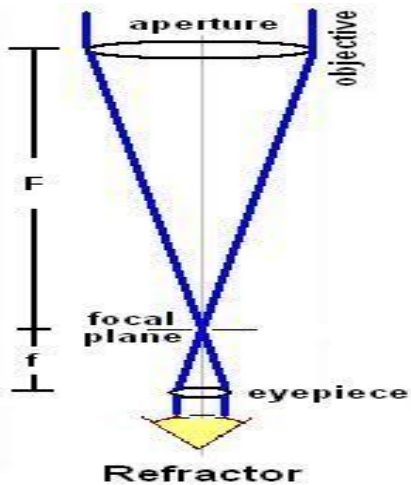
There are two basic types of telescopes: refractors and reflectors. Both have their advantages and disadvantages

Assignment:

**Find out plate scales and angular resolutions for HCT and VBT telescopes?
(use IIA web pages for the relevant information)**

Telescopes: Refractors

Refracting telescopes gather light with a lens, directing it to the eyepiece. Small scopes are often of this type, as they are simple to operate and maintain. Larger refractors, however, are very difficult to build.



The first telescope to be pointed toward the heavens by Galileo in 1609. They really have changed very little and operate by the same principle.

Telescopes: Refractors

From the very beginning, refractors suffered from a problem caused by refraction of light. Not all wavelengths, or colors, meet at the same point called chromatic aberration.

Unfortunately, focal lengths must be fairly long for all wavelengths to converge close to each other.

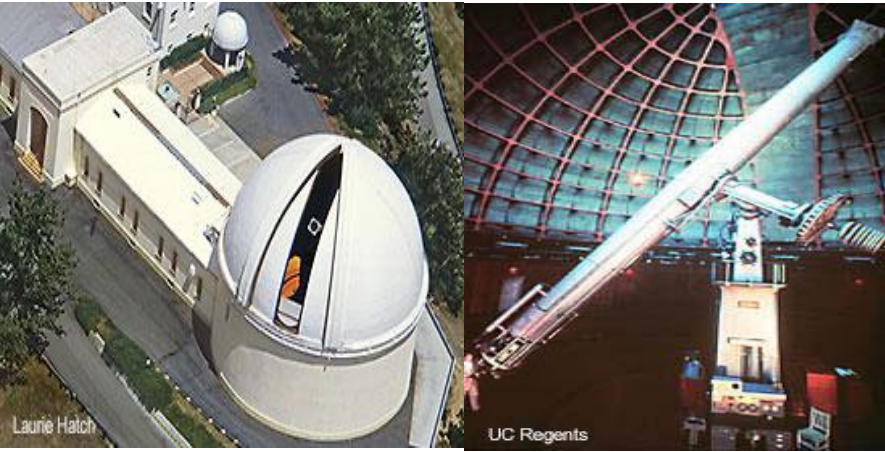
Two-element lenses, called achromats, must be figured on 4 surfaces, as opposed to one for a reflector. Refractors cost a great deal more.

They can only be supported around the edge of the lens. This limits the size of the optics.

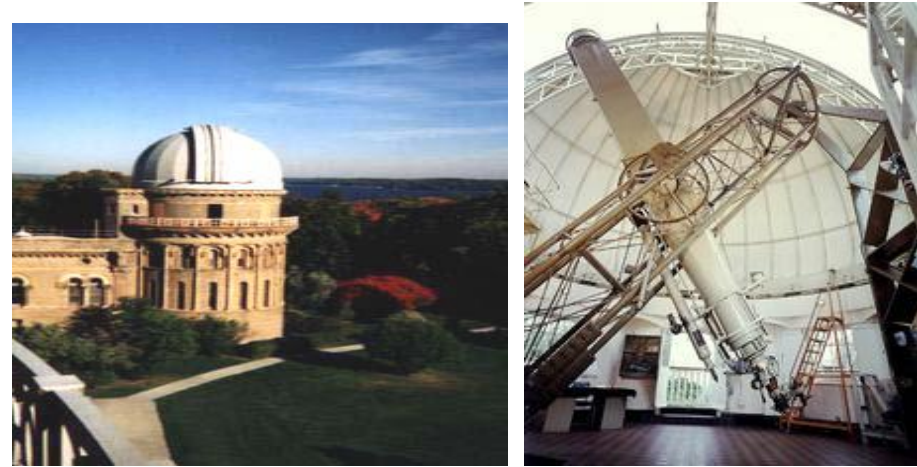
The tube length is very long.

Telescopes: Refractors

Great Lick 36-inch Refractor



40 inch-Yerkes refractor



Built during the years 1880 to 1888. When completed, the Lick Refractor was the largest refracting telescope in the world. Even today, it is second in size only to the 40-inch Yerkes Observatory refractor.

Yerkes Observatory was completed and dedicated in 1897. Credit goes to George Ellery Hale and it was funded by Charles Tyson Yerkes

Telescopes: Refractors : Chromatic aberration

Refractors

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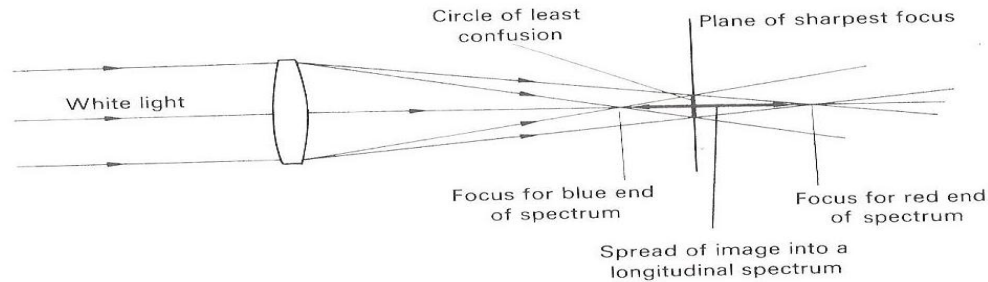


Figure 16.13. A ray diagram illustrating the effect of chromatic aberration resulting from the use of a single positive lens.

Longitudinal chromatic aberration (along the optical axis) and lateral chromatic aberration (along the focal plane)

As the light passes through the lens it suffers from chromatic aberration.

The lens-makers formula expresses focal length F as

$$\mathbf{1/F = (n-1) (1/r1 - 1/r2)}$$

$n-1$ = refractive power of the material and $r1$ and $r2$ radii of curvature

Telescopes: Refractors : Chromatic aberration

252 The optics of telescope collectors

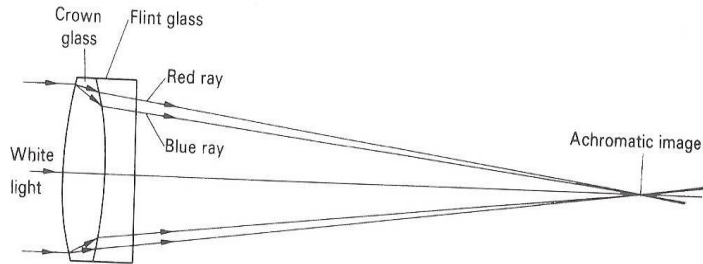


Figure 16.14. An achromatic doublet, made by cementing a positive crown glass lens to a negative flint glass lens, depicts how light rays of different colours are brought to the same focus.

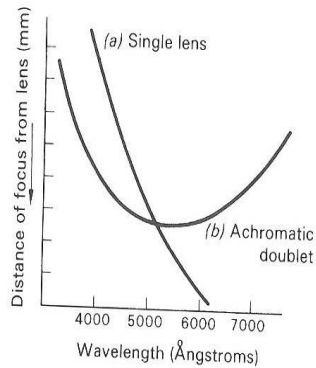


Figure 16.15. The variation of the focal length with wavelength of (a) a single lens and (b) an achromatic doublet.

Chromatic aberration can be removed by lens system by combining +ve lens with -ve lens with different (n-1) so that the dispersion cancels out.

$$F = k/(n-1); \text{ where } K = r_1 r_2 / (r_2 - r_1)$$

$$1/F = 1/F_1 + 1/F_2$$

$$F = F_1 F_2 / (F_1 + F_2)$$

By combining above equations,

$$F = k_1 k_2 / (k_2 (n_1 - 1) + k_1 (n_2 - 1))$$

Telescopes: Refractors : Chromatic aberration

The optics of telescope collectors

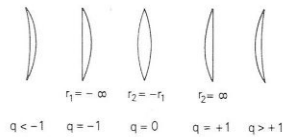


Figure 16.17. A range of lenses with different shape factors.

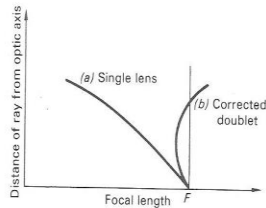


Figure 16.18. Longitudinal spherical aberration for (a) a single lens and (b) a corrected doublet

Aim of the combination is to provide a system whose focal lengths is identical in the blue and red ends of the spectrum. The achromatic condition is

$$k_1 k_2 / (k_2 (n_{1B} - 1) + k_1 (n_{2B} - 1)) = k_1 k_2 / (k_2 (n_{1R} - 1) + k_1 (n_{2R} - 1))$$

or

$$k_1 / k_2 = -n_{1B} - n_{1R} / (n_{2B} - n_{2R})$$

n_{1R} and n_{2B} are greater than n_{1B} and n_{2R} . Thus k_1/k_2 is always -ve and is achieved by combining +ve and -ve lens

Telescopes: Refractors : Spherical aberration

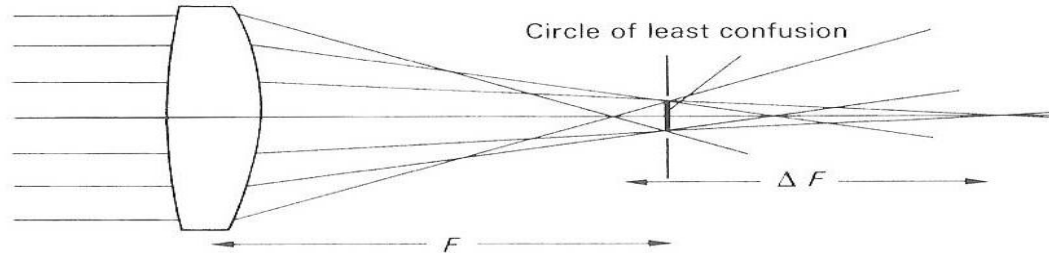


Figure 16.16. Spherical aberration produced by a single positive lens; F represents the focal length, the circle of least confusion and ΔF (exaggerated for clarity) represents the spread of the image.

Amount of spherical aberration depends on the shape of a lens

The shape factor,

$$q = r_2 + r_1/r_2 - r_1$$

Where, r_2 and r_1 are radii of the two lens surface.

It is found that spherical aberration is minimum when q is close to $+0.7$

Telescopes: Refractors : Spherical aberration

The optics of telescope collectors

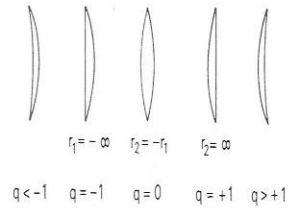


Figure 16.17. A range of lenses with different shape factors.

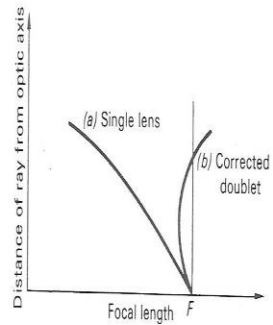


Figure 16.18. Longitudinal spherical aberration for (a) a single lens and (b) a corrected doublet.

Telescopes: Refractors : Coma

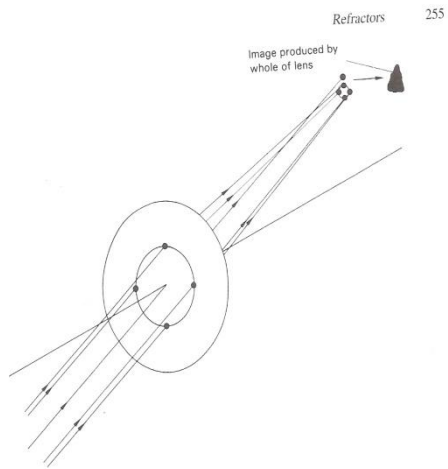


Figure 16.19. When coma is present, any annulus of the lens produces an annular image; the total aberrated image can be thought of as being made up of a series of such annular images, the sizes increasing as the outer zones of the lens are considered.

Telescopes: Reflectors

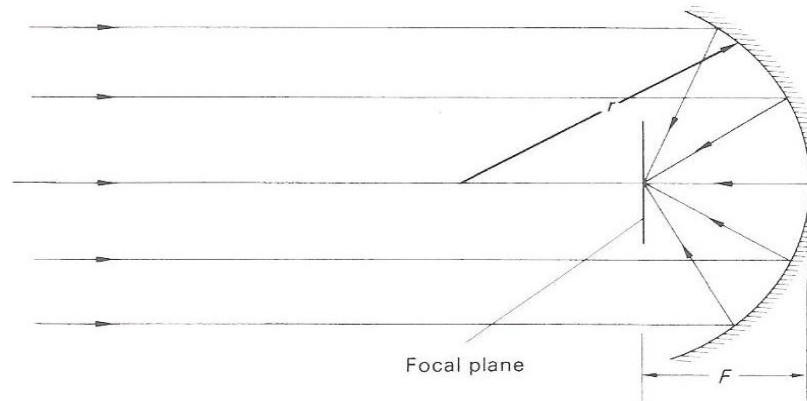


Figure 16.22. Simple ray diagram illustrating the ability of a spherical concave mirror to act as an image form (All rays are paraxial.)

A spherical concave mirror of radius, r , has a focal point at a distance equal to $r/2$ from the mirror surface. Images from distant objects form in a plane at right angles to the optics axis.

In order to have access to the primary image, the central part of mirror surface will be ineffective. This gave rise two basic forms of reflectors : **Newtonian and Cassegrain systems**

Telescopes: Reflectors

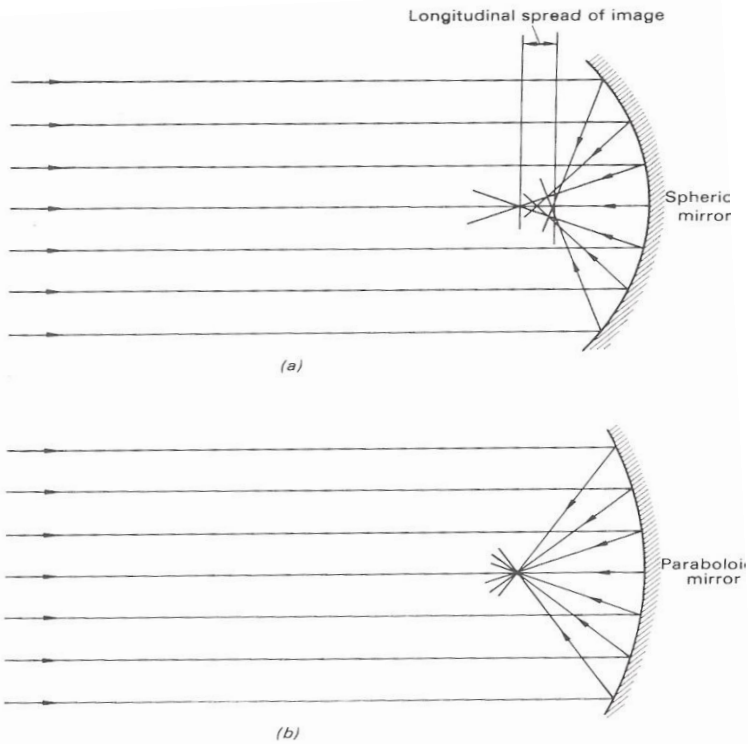


Figure 16.23. (a) Spherical mirror exhibiting the effect of spherical aberration. (b) Paraboloidal mirror; incident rays parallel to the optic axis are brought to the same focus, independent of distance from the axis.

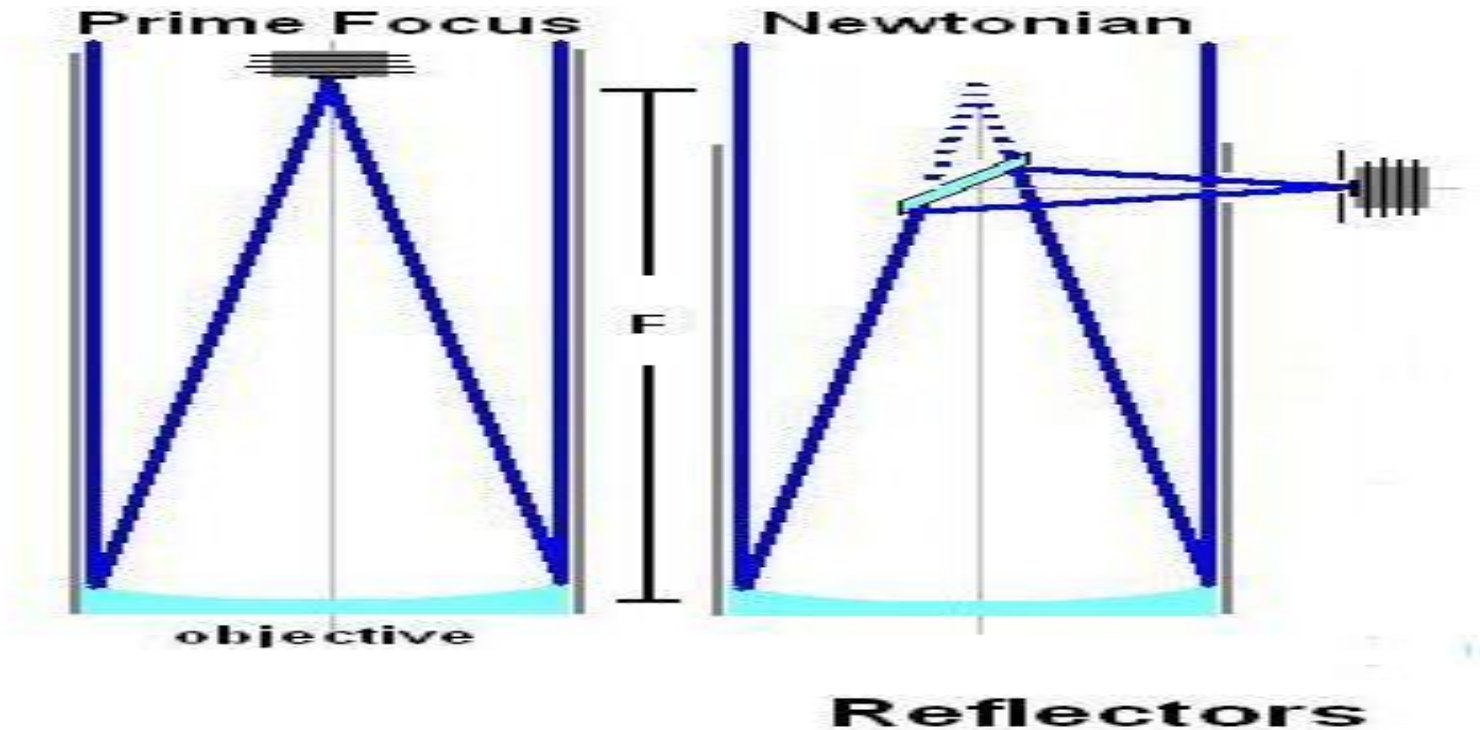
Spherical aberration

Assignment:

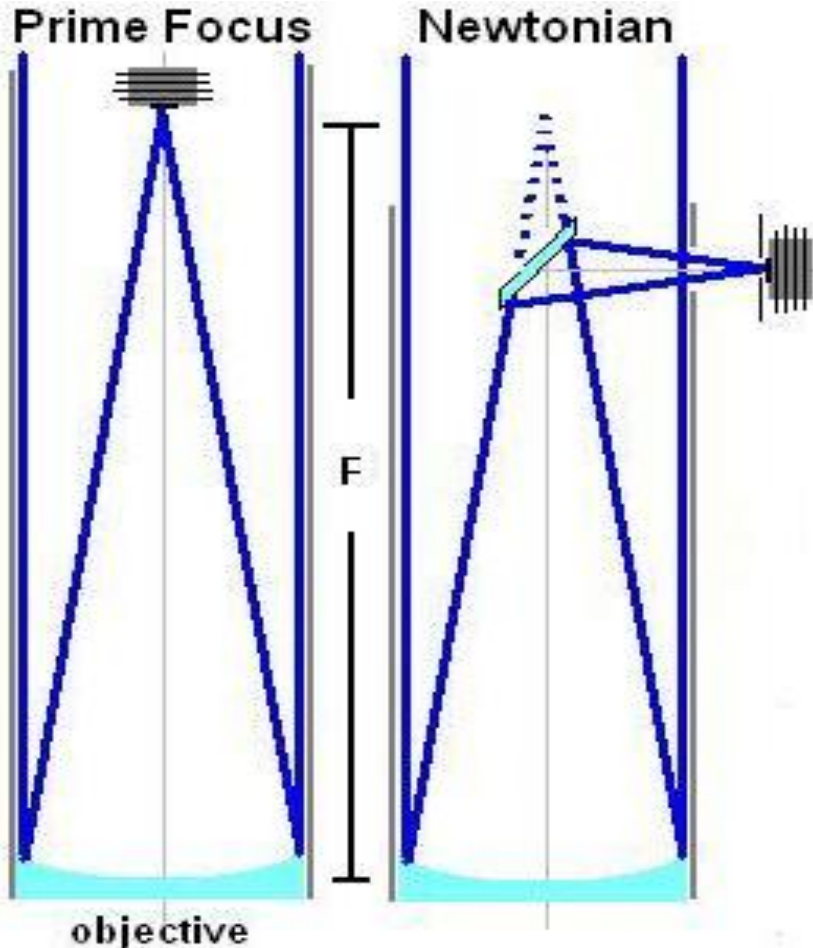
make a study on parabolic and spherical mirrors. Discuss their Pros and cons in astronomy (use any available source).

Telescopes: Reflectors: Newtonian

The simplest type of reflecting telescope employing a concave parabolic primary mirror. The light reflects off the mirror and comes to a focus on-axis. A small flat mirror, called the Newtonian Diagonal, is placed before the focus to direct the beam to the side where it can be readily examined. The size of the diagonal mirror increases as the focal ratio of the optic becomes faster.



Telescopes: Reflectors: Newtonian

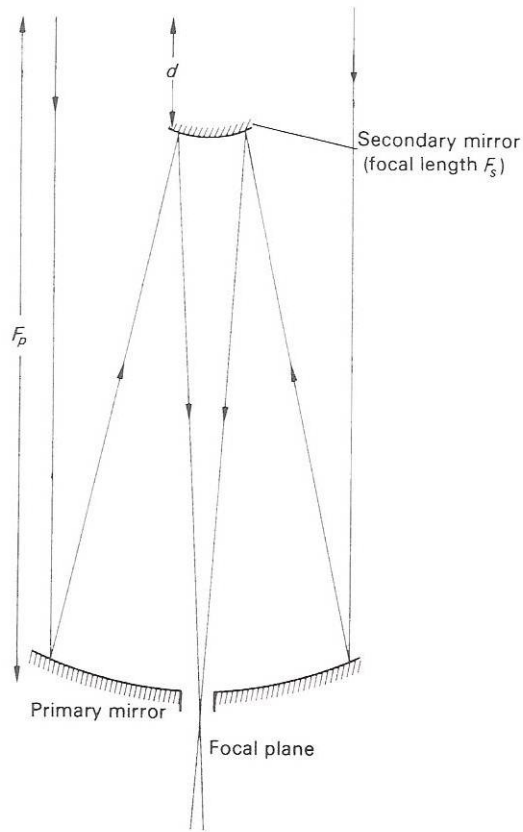


- a) The tube length is very long.
- b) The field is very poorly corrected, especially for fast focal ratios.
- c) For visual observations the observer must climb tall ladders to reach the eyepiece.
- d) For electronic detectors the equipment must be placed far from the telescope axes and longer cable lengths are required.
- e) Heat from the equipment or observer is likely to degrade the image quality.

For these reasons the Newtonian is a poor choice for a modern research instrument or public outreach facility

Telescopes: Reflectors: The Cassegrain system

The optics of telescope collectors



The Cassegrain is the most common system for the modern observatory. **The flat Newtonian diagonal is replaced with a secondary mirror with a convex surface.** Light is reflected back through a hole in the primary mirror.

A Cassegrain telescope has advantages:

- a) The tube length is compact.
- b) **The focal plane and hence instrumentation is readily accessible**

Telescopes: Reflectors: The Schmidt telescope

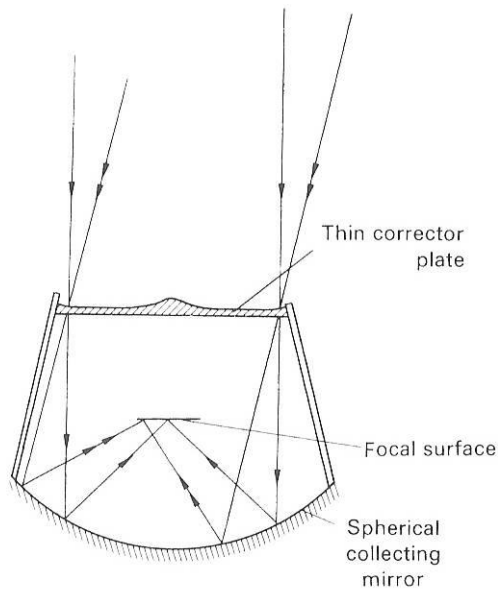


Figure 20.7. A Schmidt telescope or camera.

The UK Schmidt Telescope (UKST) is a special purpose camera, a survey telescope with a very wide-angle field of view. It was designed to photograph 6.6 x 6.6 degree areas of the night sky on photographic plates 356 x 356 mm (14 x 14 inches) square.

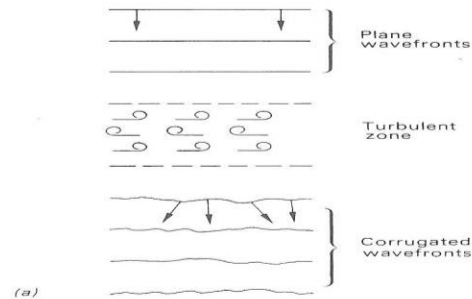
From 1973 to 1988, the UKST was operated by the Royal Observatory, Edinburgh .



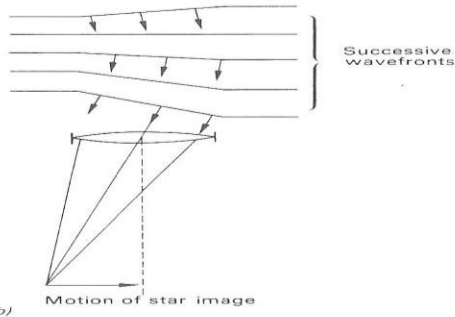
Astronomical seeing

Earth's atmosphere is turbulent due to airflows. As a result wavefronts thru the atmosphere are subjected to the random phase delays.

ronomical optical measurements



(a)



(b)

The effect of atmospheric turbulence. (b) The dancing effect of a star i

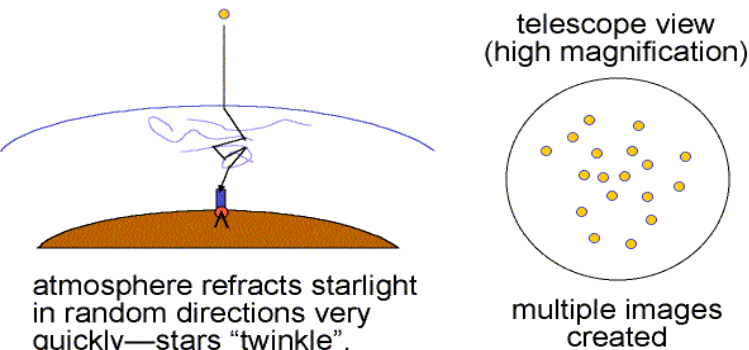
Two consequences: intensity scintillation seeing

Minute temperature differences between the individual pockets of air within the atmosphere introduce small **differences in refractive index** resulting wavefronts get corrugated

The typical height at which the deforming taking is given by

$$\underline{r_0/\alpha = 100/3 \times 206265 = 7 \text{ km}}$$

α is angular size of corrugation and r_0 is the physical size (found from experiments)



atmosphere refracts starlight in random directions very quickly—stars “twinkle”.

multiple images created

Adaptive optics

AO provides a clearer view of the universe by compensating for atmospheric turbulence that causes stars to "twinkle." **The adaptive optics adjustment is made with a deformable mirror that is almost infinitely adjustable. It changes shape in numerous places hundreds of times per second, compensating for changing atmospheric conditions to focus light precisely.** Essentially, when a celestial object is to be observed, a fairly bright star nearby is monitored, and a correction is made for the "twinkle" that is observed. This correction is then applied to the object when it is observed.

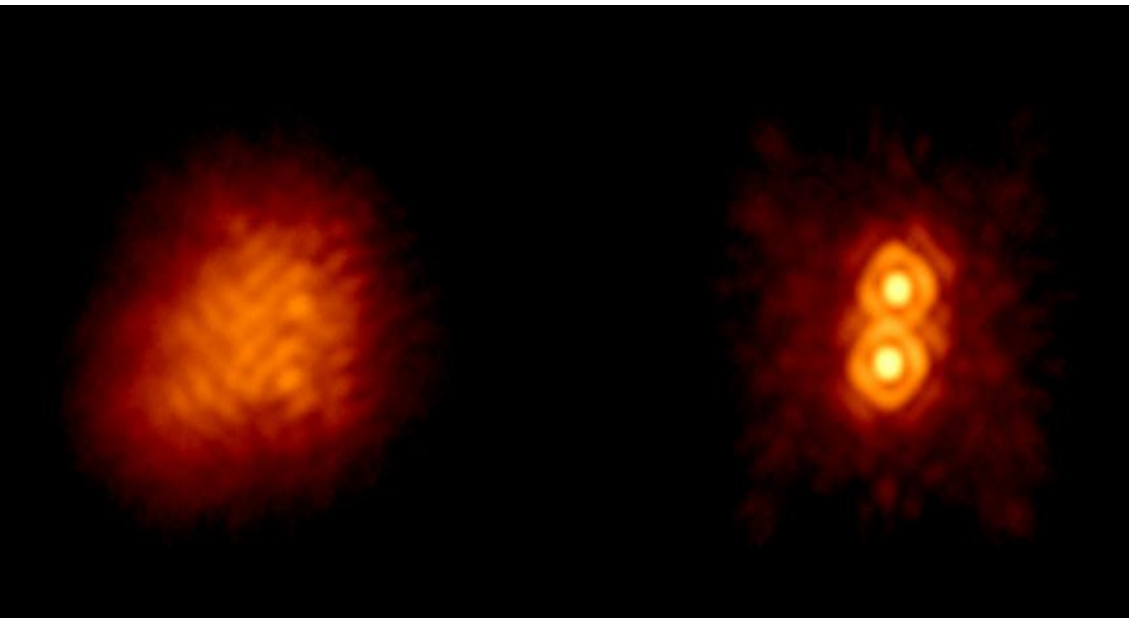
Adaptive optics- laser guided star

. They shine a narrow sodium laser beam up through the atmosphere. At an altitude of about 60 miles, the laser beam makes a small amount of sodium gas glow. The reflected glow from the glowing gas serves as the artificial guide star for the adaptive-optics system.



Adaptive optics

Adaptive optics is a technique that allows ground-based telescopes to remove the blurring affects caused by Earth's atmosphere. The image below shows the dramatic improvement gained through the use of adaptive optics.



The binary star IW Tau is revealed through adaptive optics. The stars have a 0.3 arc second separation. the images

Adaptive optics

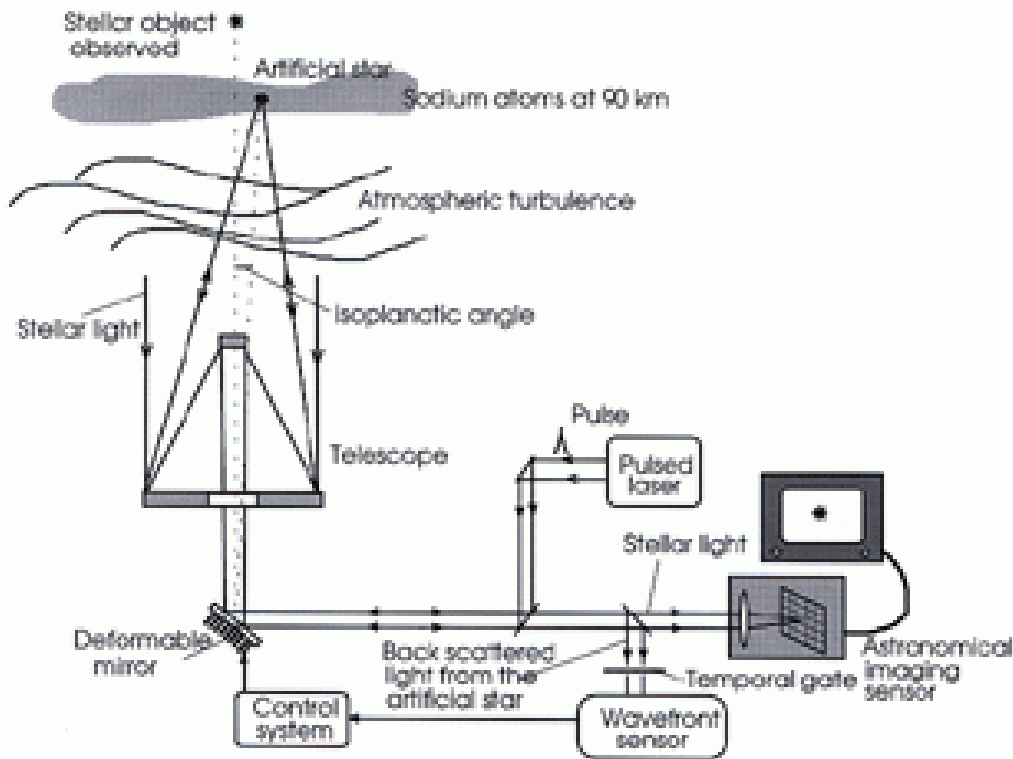
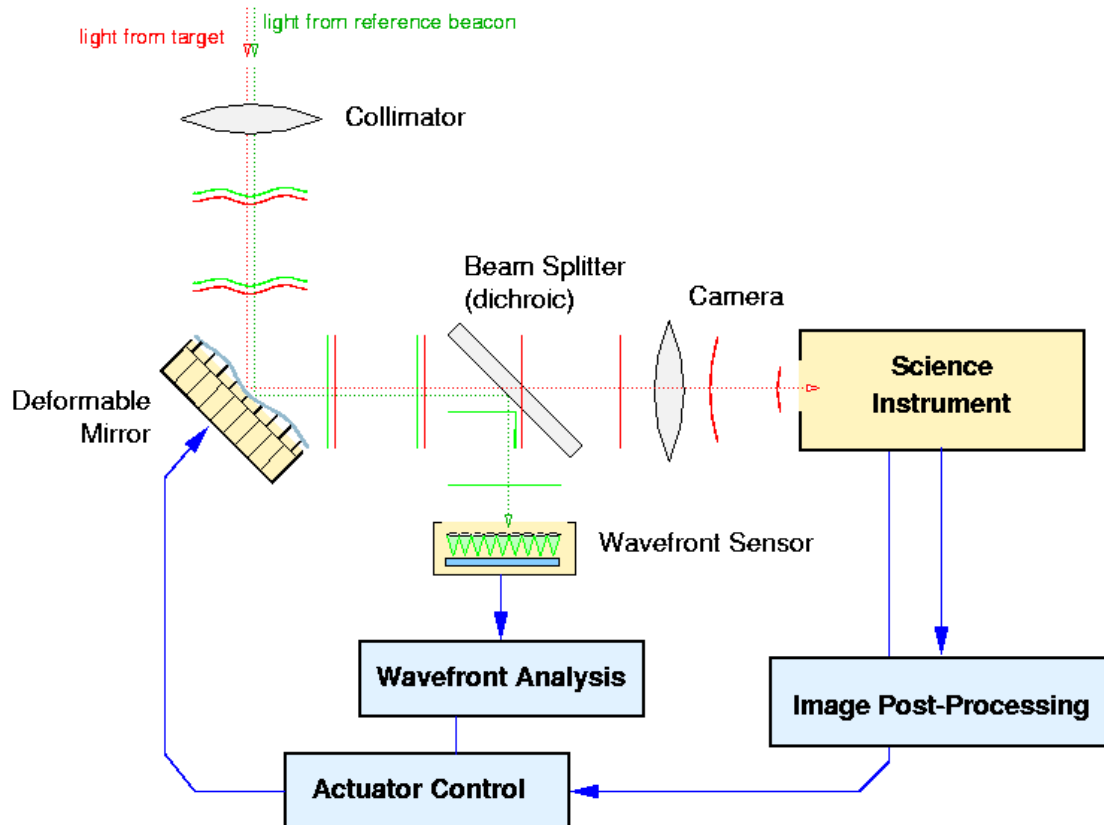
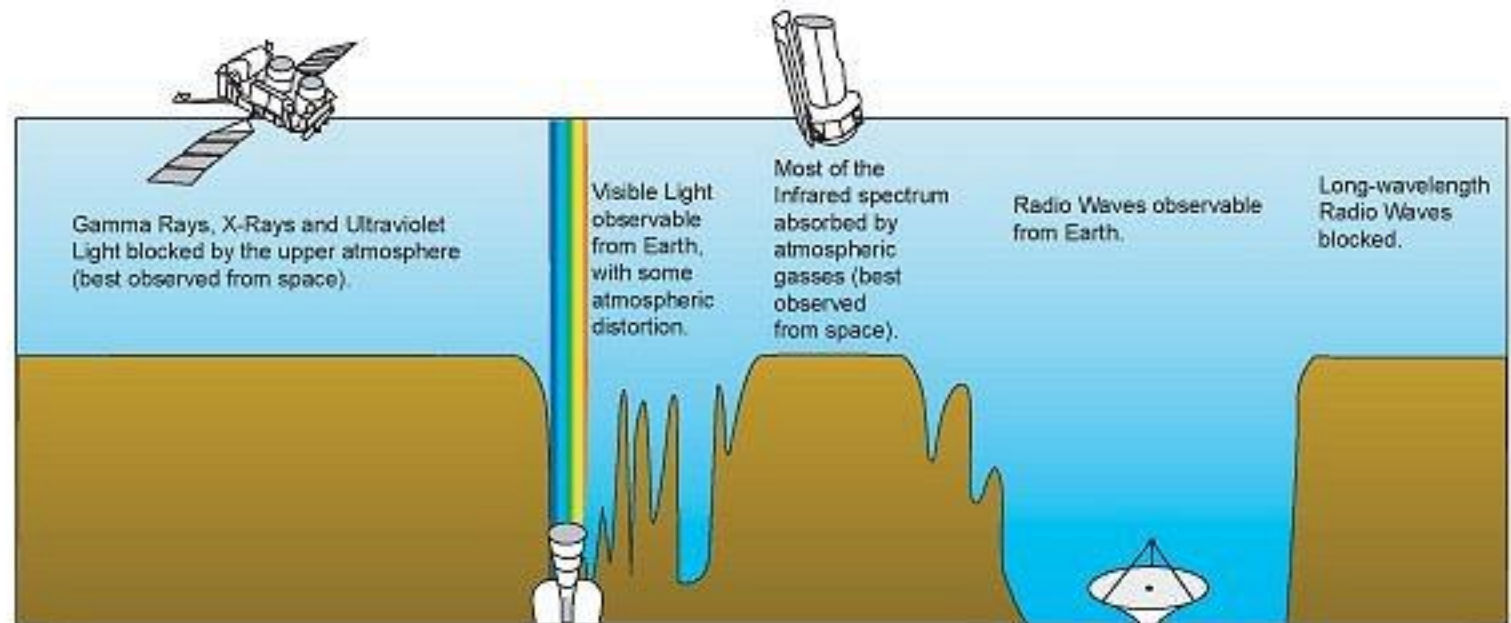
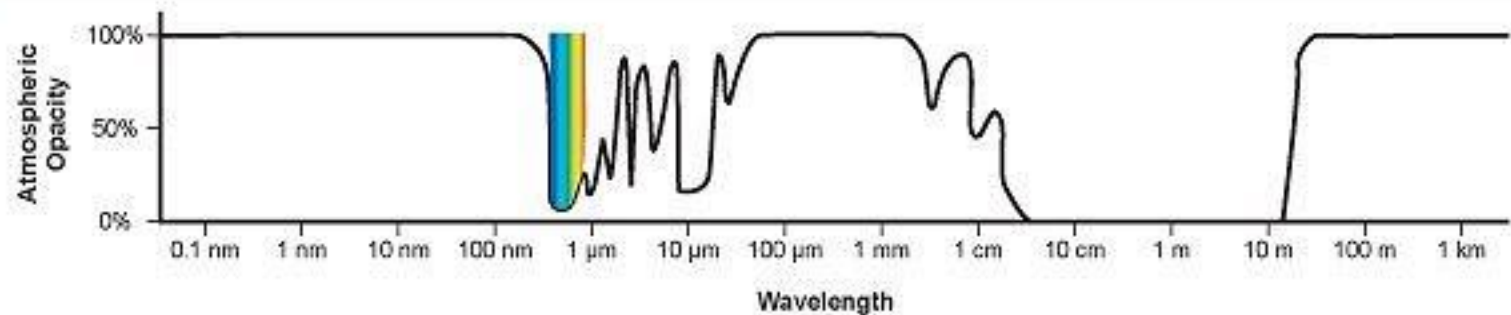


Figure 4: Adaptive Optics with laser guide star

Adaptive optics





Telescopes: Mountings

The classical mounting for an astronomical telescope is to have an axis parallel to the Earth's north-south axis, called an equatorial mounting.

Another popular mounting is Altazimuth mounting which requires track the star on two axes with varying drive rates. Now that computer controlled drive systems can be made which allow constantly varying drive rates to be used on two axes..

Mountings: Equatorial

The telescope moves north-south about the declination axis and east-west about the polar axis.

To *point* at a target requires moving the telescope about both axes. To *track* a target, however, requires movement about the polar axis only, at the same rate that Earth spins.

chief advantage of the equatorial mounting: the N-S position doesn't change, and a single drive can regulate the E-W tracking.

main drawback is that the polar axis is difficult to orientate with respect to the ground, and it is different for every observatory.

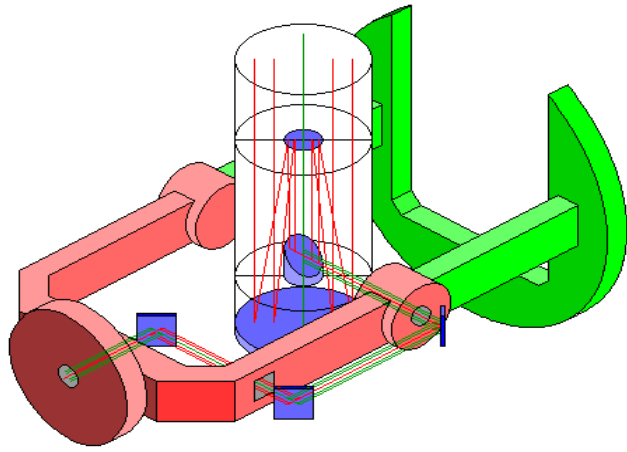
Positioning the mounting at odd angles creates difficulties that increase rapidly as the size and mass of the telescope increase.

Horseshoe mounting (equatorial)

The **horseshoe** mount is a design that avoids the limitation of the yoke with respect to objects near the pole. It involves mounting the telescope in a frame like this:

2.3m vbt

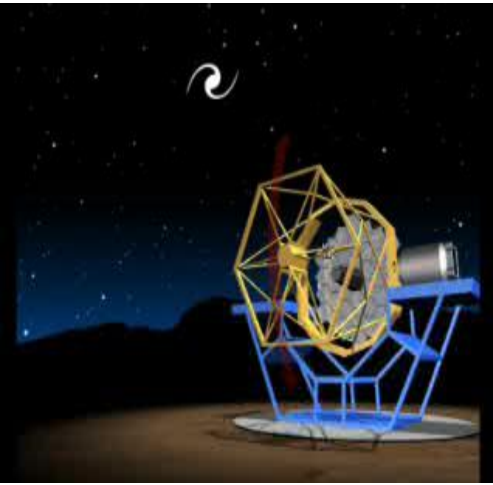
5m Hale at Palomer



Telescopes-Mounts: Altazimuth

WHT 4.2m canary islands, Spain

HCT 2.0m, Hanle, India

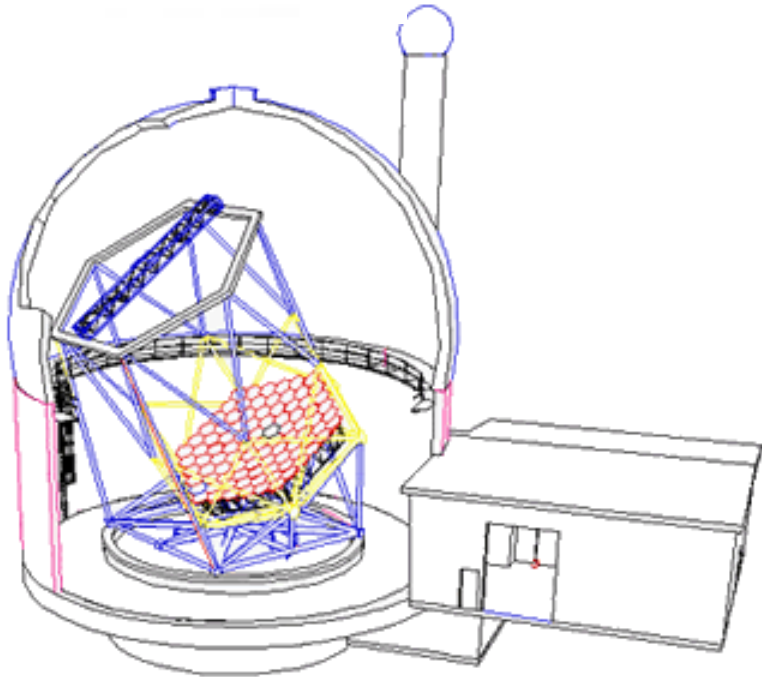


A few modern telescopes (HET)

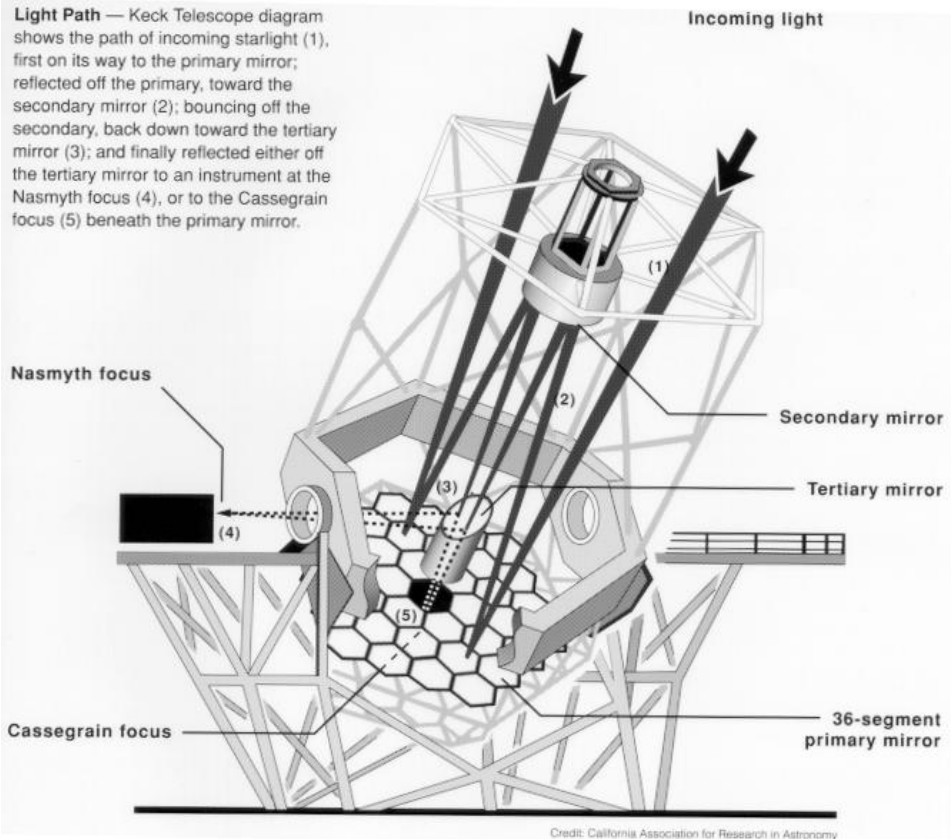


The HET at Mount Fowlkes

10.0 m telescope

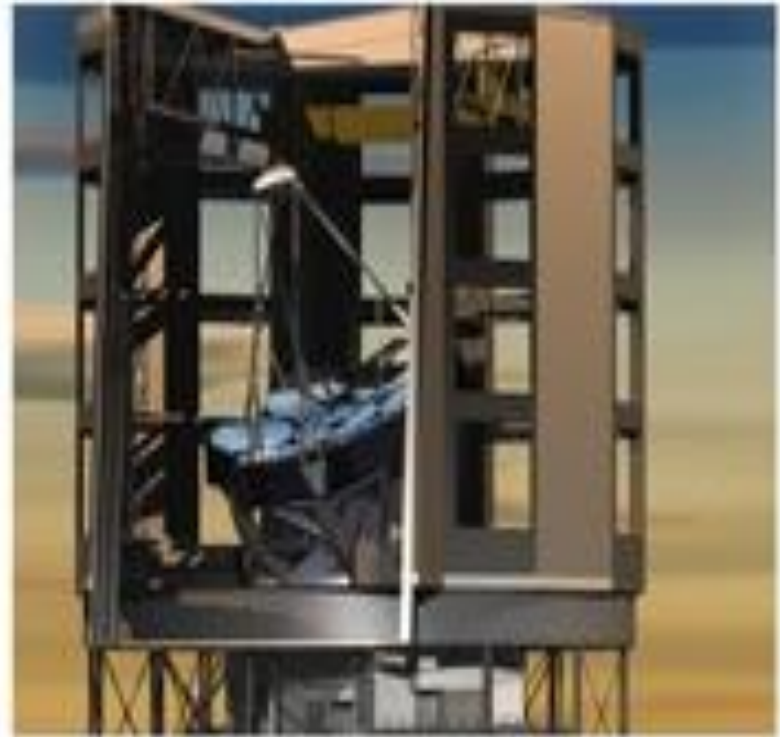
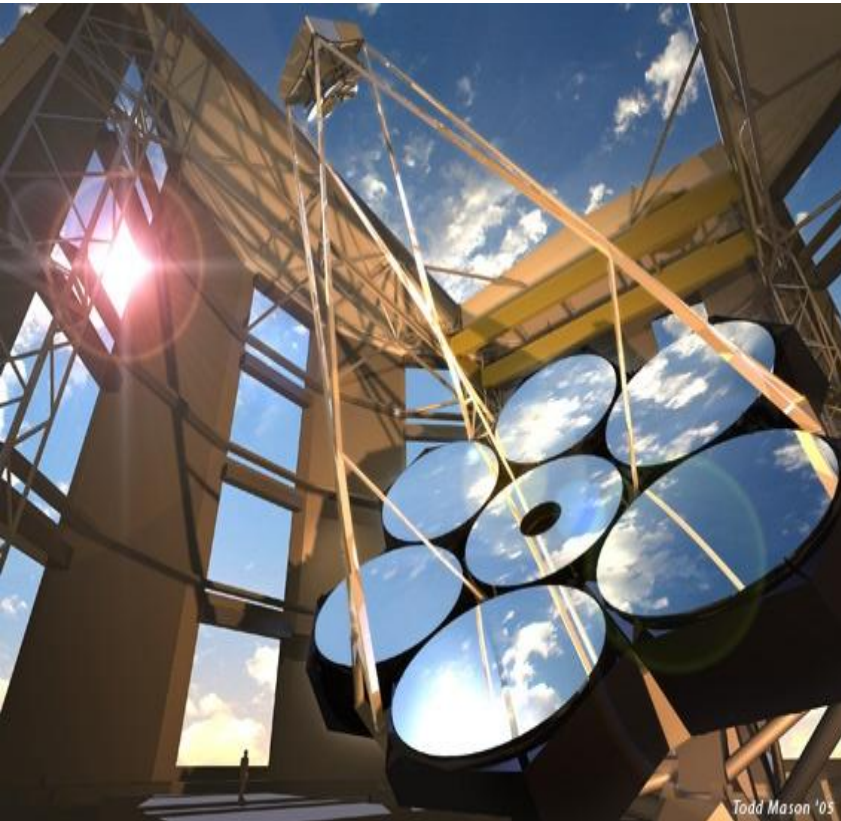


A few modern telescopes (keck)



A few modern telescopes (GMT)

~26 m and projected completion in 2016



A few modern telescopes (OWL)

ESA's overwhelmingly large telescope (100m)

